Coherent propagation of a heton near a submerged cylinder in a two layer fluid

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Problem formulation

Motivation

- Vortices are omnipresent in the ocean playing an important role in generating synoptic and mesoscale dynamics. Strong vortices are often observed near bottom irregularities. Therefore, it is naturally that problems concerning vortices interacting with topography have been attracting a lot of attention. Different approaches are exploited to get insights into such interactions.
- The point vortex model presents possibly the simplest and most accessible approach, allowing one to formulate closed ordinary differential equation systems that govern the point-vortex dynamics. These systems can vary in complexity given varied vortex interactions in question.
- Another approach represents a vortex structure as a closed region with some constant distributed vorticity comprised within it. Then, contours comprising different vorticities evolve according to the vorticity equation.The contours can merge or decompose creating new contours.

Problem formulation

Configuration in question

- We study a compensated two-layer vortex pair, which is conventionally known as a heton, encountering a submerged bottom cylindrical feature.
- First, we analyze the evolution of an unstable heton with the use of the contour dynamics technique. This case has no analogy in the point vortex model.
- Then, given a stable heton, we delineate typical regimes of the system dynamics with the use of the point vortex model and then compare it to the similar dynamics observable with the help of the contour dynamics techniques

Problem formulation

Configuration in question

We assume the quasi-geostrophic, rigid lid, f-plane approximations applying for a fluid flow with two layers of constant densities. The law of potential vorticity conservation is valid in each layer,

$$\frac{D_i q_t}{Dt} = 0, \tag{1}$$

with

$$q_1 = \Delta\psi_1 + \frac{f}{H_1}\zeta, \quad q_2 = \Delta\psi_2 + \frac{f}{H_2}\left(\tilde{h} - \zeta\right), \tag{2}$$

where Δ is the two-dimensional Laplace operator, ψ_i is the stream-function equal to the pressure anomaly p_i in the i-th layer ($i = 1$ corresponds to the upper layer) of the depth H_i and density ρ_i, f is the Coriolis parameter, $\tilde{h}$ is the bottom perturbation, and $\zeta = \frac{f}{g^*}\left(\psi_2 - \psi_1\right)$ is the perturbation of the interface between the layers with $g^* = g\frac{(\rho_2 - \rho_1)}{\rho_2}$ being the reduced gravity acceleration.

Problem formulation

Configuration in question

The stream-functions ψ_i can be represented through two auxiliary stream functions: barotropic Ψ and baroclinic Ψ' as follows

$$\psi_1 = \Psi - \frac{H_2}{H}\Psi',\ \psi_2 = \Psi + \frac{H_1}{H}\Psi', \tag{3}$$

where

$$H\Delta\Psi = H_1 q_1 + H_2 q_2 - f\tilde{h},\ \ \Delta\Psi' - k_1^2\Psi' = q_2 - q_1 - f\frac{\tilde{h}}{H_2}, \tag{4}$$

and $k_1 = \frac{1}{L_D} = f\left(\frac{H}{g^* H_1 H_2}\right)^{1/2}$, $H = H_1 + H_2$ is the total depth, and L_D is the internal Rossby radius of deformation.

Problem formulation

Configuration in question

In this paper, we are interested in a bottom perturbation due to an isolated cylindrical feature of a constant radius a, and constant height h, i.e. $\tilde{h} = \begin{cases} h, \ r \le a, \\ 0, \ r > a, \end{cases}$ located in the bottom layer of the two-layer flow. The corresponding barotropic and baroclinic stream functions comply with

$$H \Delta \Psi_0 = -fh, \ \ \Delta {\Psi_0}' - k_1^2 {\Psi_0}' = -\frac{fh}{H_2}. \tag{5}$$

Hence the solutions

$$\Psi_0 = -\frac{fha^2}{4H} \begin{cases} \left(\frac{r}{a}\right)^2, \ r \le a, \\ 1 + 2\log\frac{r}{a}, \ r > a, \end{cases}$$
$${\Psi_0}' = -\frac{fh^2}{k_1^2 H_2} \begin{cases} 1 - ak_1 I_0\left(k_1 r\right) K_1\left(ak_1\right), \ r \le a, \\ ak_1 I_1\left(ak_1\right) K_0\left(k_1 r\right), \ r > a. \end{cases} \tag{6}$$

Problem formulation

Finite-core vortex patches

We distribute two initially circular patches with constant vorticities Π_i in each layer such that the vorticity from the upper layer is compensated by the vorticity in the lower layer, i.e. $H_1\Pi_1 + H_2\Pi_2 = 0$. Thus, the total potential vorticity of the two-vortex structure is equal to zero.

Problem formulation

Point-vortex model

Here, we introduce the vortex component by defining point vortex perturbations in the layers. We consider the following two-layer vortex structure: one point vortex of strength μ_1 arranged in the upper layer, and the other one with strength μ_2 in the lower layer, so

$$q_i = f\mu_i L_0^2 \delta\left(x - x_i\right)\delta\left(y - y_i\right), \tag{7}$$

where $\delta\left(\cdot\right)$ is the Dirac delta, x, y are the Cartesian coordinates, and x_i, y_i are the i-th vortex's coordinates, and L_0 is a characteristic horizontal scale.

Problem formulation

Point-vortex model

Governing equations for the vortex trajectories ensue

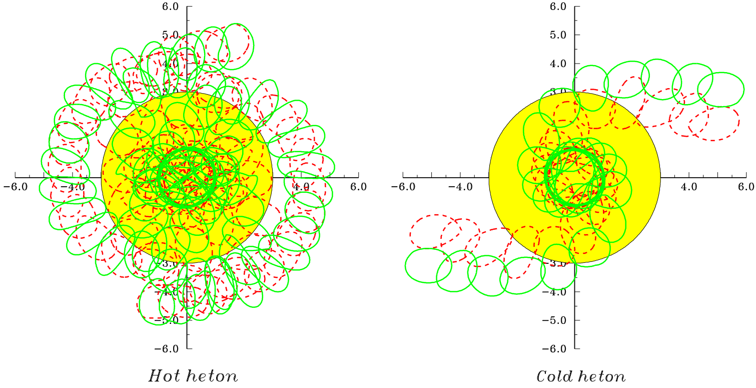
$$\begin{aligned} \frac{dx_\alpha}{dt} &= -\frac{\gamma^2}{H}\frac{\mu_{3-\alpha}H_{3-\alpha}}{r_{12}}(y_\alpha - y_{3-\alpha})\left[\frac{1}{r_{12}} - K_1(r_{12})\right] + \\ &\quad \frac{h}{H} y_\alpha \Phi_\alpha(r_\alpha) \\ \frac{dy_\alpha}{dt} &= \frac{\gamma^2}{H}\frac{\mu_{3-\alpha}H_{3-\alpha}}{r_{12}}(x_\alpha - x_{3-\alpha})\left[\frac{1}{r_{12}} - K_1(r_{12})\right] - \\ &\quad \frac{h}{H} x_\alpha \Phi_\alpha(r_\alpha), \end{aligned} \tag{8}$$

where $r_\alpha = \left(x_\alpha^2 + y_\alpha^2\right)^{1/2}$, $\alpha = 1, 2$, $r_{12} = \left((x_1 - x_2)^2 + (y_1 - y_2)^2\right)^{1/2}$.

Unstable heton

Finite-core vorticity patches

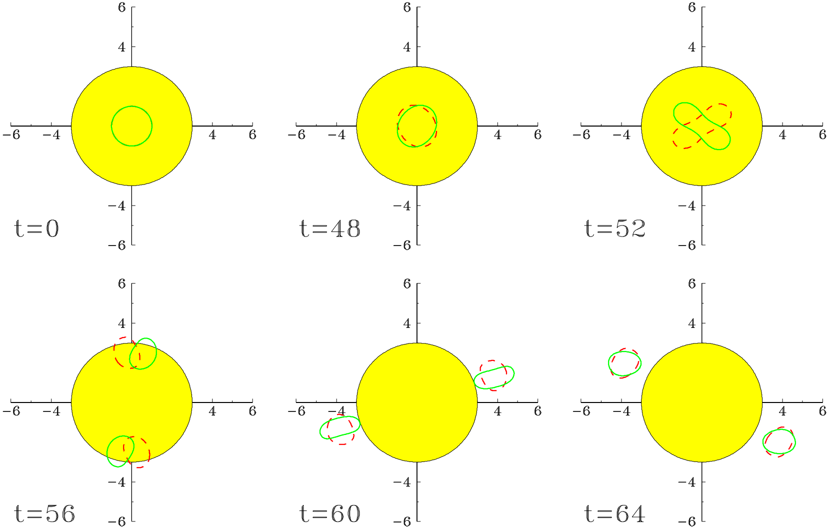
We consider an aligned heton. For weak stratification, for instance $\gamma = 1.8$, there appears an unstable azimuthal mode featuring two separate dominant vorticity patches in each layers. Because of the sufficient nonlinearity, two distinct two-layer vortex pairs form.



Unstable heton

Finite-core vorticity patches

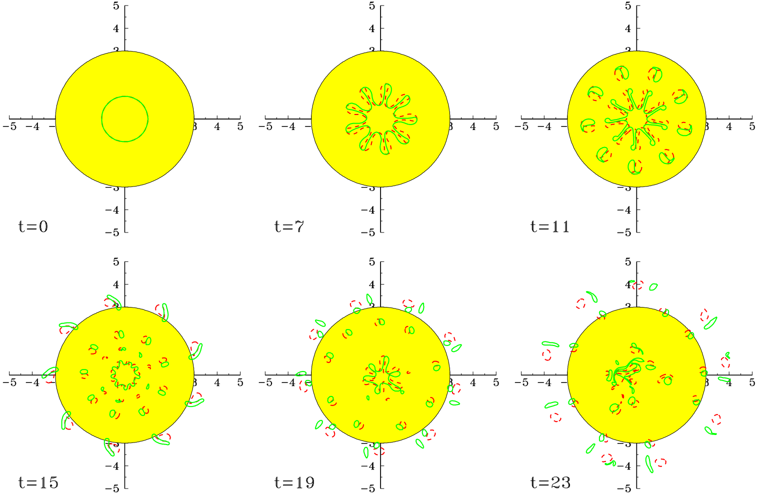
Detailed evolution of a hot heton.



Unstable heton

Finite-core vorticity patches

If stratification is weaker, the effect also becomes more subtle ($\gamma = 14$).



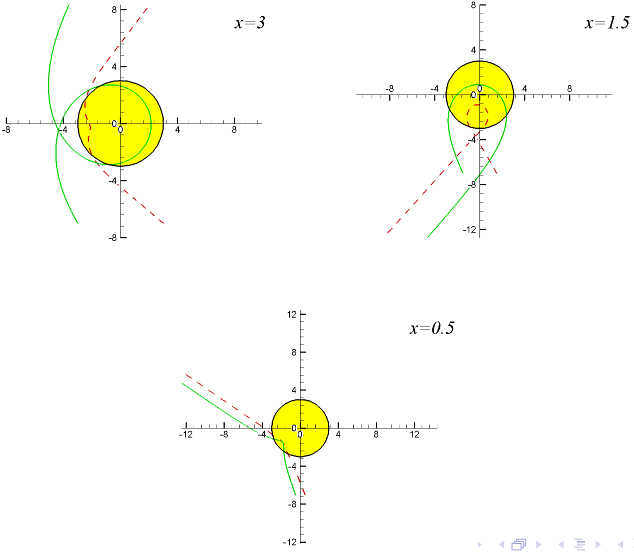
Stable heton – Unbounded dynamics

Point-vortex model

To obtain similar regimes of motion in the both approaches, one needs to strengthen stratification such that a heton becomes stable. To start, we deal with the point-vortex system. The physical configuration $H_1 = H_2 = 1/2, h = 0.1, R = 3$. The figures depict vortex trajectories for different initial distances between the vortices as $\mu_1 = -\mu_2 = \mu = -21$, $y_1 = y_2 = y = -7$, $x_1 = -x_2 = x$.

Stable heton – Unbounded dynamics

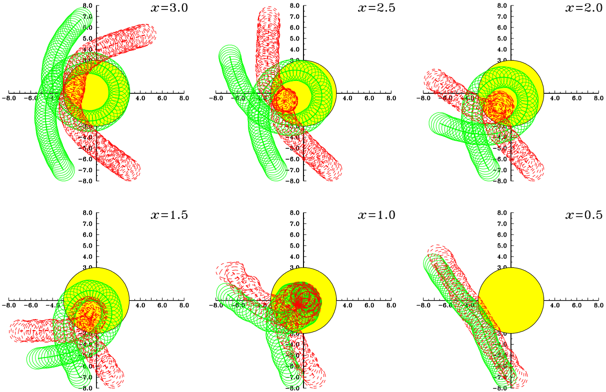
Point-vortex model



Stable heton – Unbounded dynamics

Finite-core vorticity patches

Finite-core vorticity patch model. Instantaneous contours of a cold heton consisted of initially circular vortex patches of unit radii as $\gamma = 1, y = -7$ for given x. The consequent contours are depicted over a unit time interval.



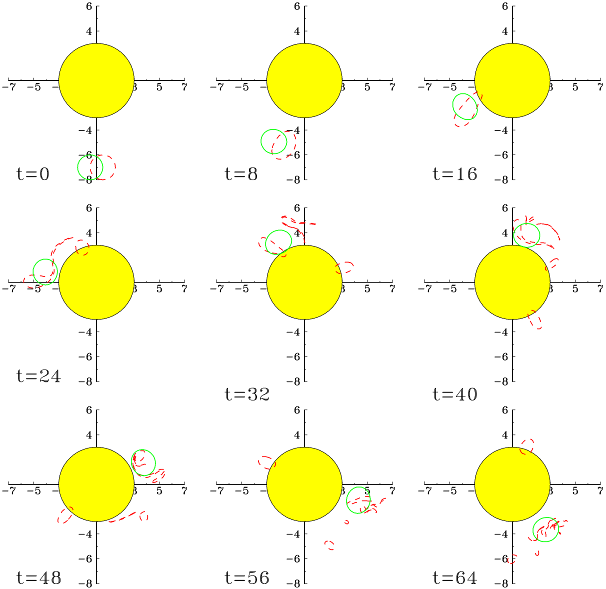
Unstable heton – Unbounded dynamics

Finite-core vorticity patches

However, if the vorticity patches are less stable, the point vortex model cannot reflect the dynamics correctly. The next figure demonstrates an example of complex dynamics of the vorticity patches with the same initial conditions , except that the potential vorticity of the heton's vortices is decreased by half.

Unstable heton – Unbounded dynamics

Finite-core vorticity patches



Stable heton – Bounded dynamics

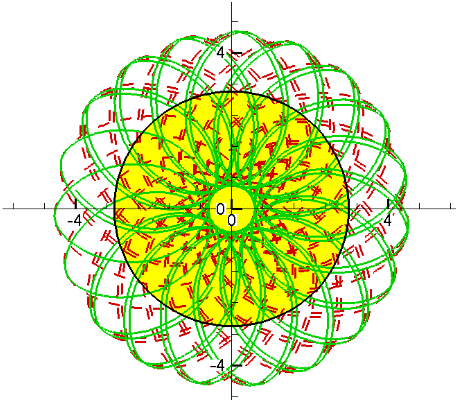
Point-vortex model

The previous section is primarily concerned with the unbounded dynamics of the two-layer self-propagating vortex structure. This dynamics is intrinsic to both the finite-core vorticity patch model, and the point-vortex model. The finite-core vorticity patches, if surviving the interaction with the cylindrical obstacle, can behave qualitatively very similar to the point-vortices. When the vorticity patches undergo significant deformation and redistribution of vorticity, they cannot anymore sustain their self-propulsion, which leads to them being trapped by the topography. This dynamics is clearly not present in the point-vortex model. However, a regime of entrapping of both vortices without being significantly deformed is possible in the both models under consideration. In this section, such a type of the heton dynamics near the topographic feature is under study.

Stable heton – Bounded dynamics

Point-vortex model

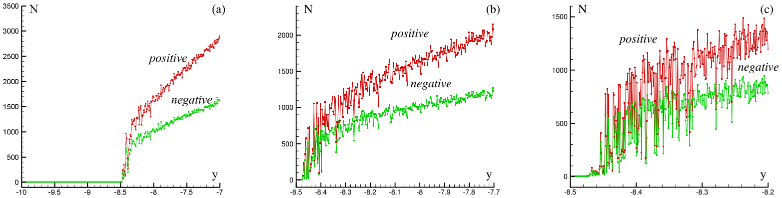
The bounded motion of a point vortex heton with a symmetrical initial configuration. The solid (dashed) trajectory corresponds to the upper-layer (lower-layer) vortex. $\mu = 21$, $y = -2$, $x = -0.5$.



Stable heton – Bounded dynamics

Point-vortex model

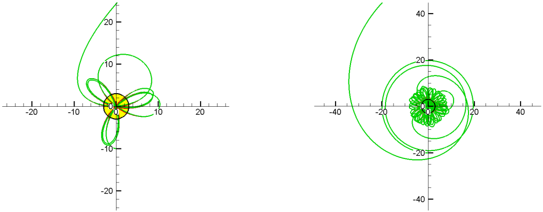
The number of intersections between vortex trajectories (for the positive μ_1 and negative point vortex μ_2 of the heton) and the y-axis depending on the starting y-coordinate as $\mu = 21$, $x = 0.01$. 300 evenly distributed initial conditions over a different interval on the y-axis: (a) $y \in [-10, -7]$; (b) $y \in [-8.5, -7.6]$; (c) $y \in [-8.5, -8.2]$.



Aligned heton – Mixed dynamics

Point-vortex model

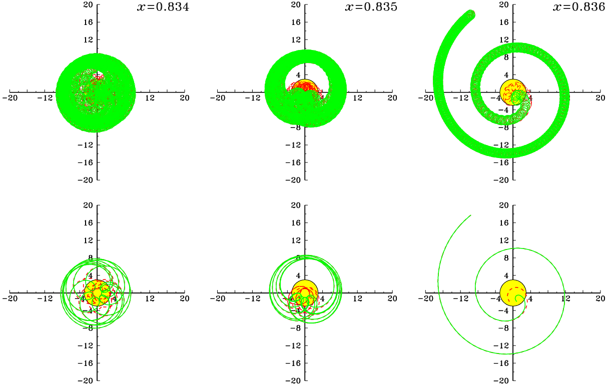
Examples of an aligned heton's trajectories demonstrating the irregular behavior. Different initial conditions result in generally unpredictable shapes of heton trajectories. (a) $\mu = 300$, $y = -10$; (b) $\mu = 300$, $y = -20$.



Aligned heton – Mixed dynamics

Finite-core vorticity patches

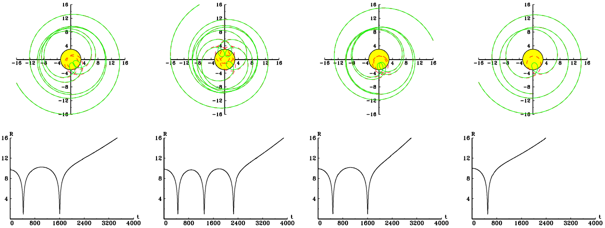
Now, let us demonstrate the analogous dynamics realized in the continuous vortex model.



Aligned heton – Mixed dynamics

Finite-core vorticity patches

The upper row shows the trajectories of the vorticity patches' centers as $x = 0$ and given y. The lower one shows the distances between the centers of vorticity of the upper layer vortex patches and the obstacle's center.



y=9.7 y=9.8 y=9.9 y=10.0

Thank you very much for your attention!
Many thanks to the organizing committee!