

Third International Workshop  
NONLINEAR PROCESSES IN OCEANIC AND ATMOSPHERIC FLOWS

Evolution of secondary whirls in  
thermoconvective vortices in a route  
to chaos

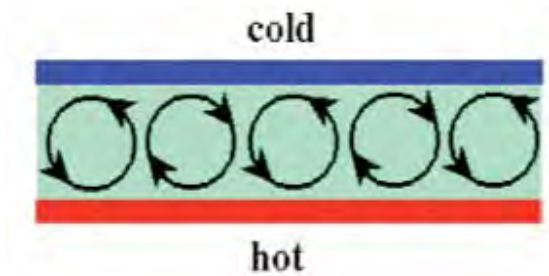
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Dpto. Matemáticas, Universidad de Castilla-La Mancha, Spain  
(María Cruz Navarro, Damián Castaño UCLM)

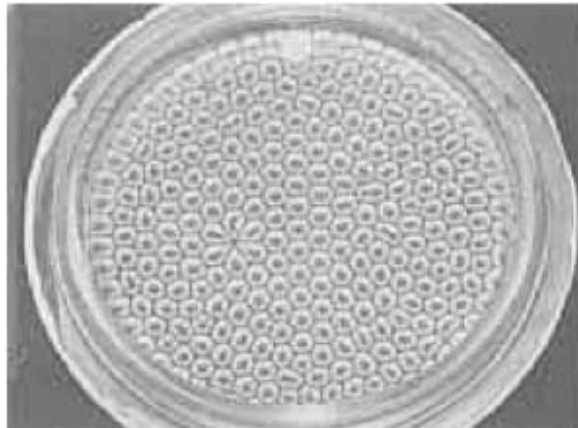


## Background

### Rayleigh-Bénard convection

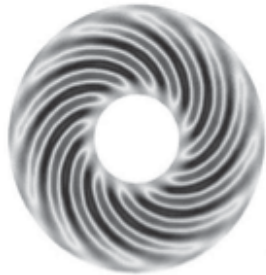


*H. Bénard, 1900*

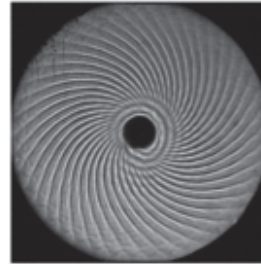


## Background

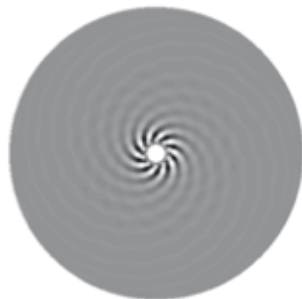
### Localized heating in a cylindrical annulus and small cell



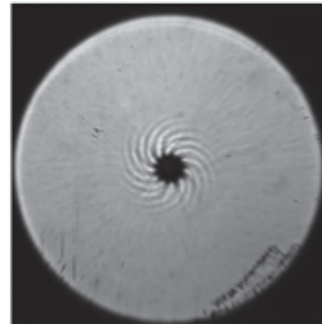
S. Hoyas, H. Herrero y A. M. Mancho, *Phys. Fluids* (2005)



N. Garnier y A. Chiffaudel, *Eur. Phys. J.* (2001)



S. Hoyas, et al. *Phys. of Fluids* (2005)



Waves,  
mass transport?



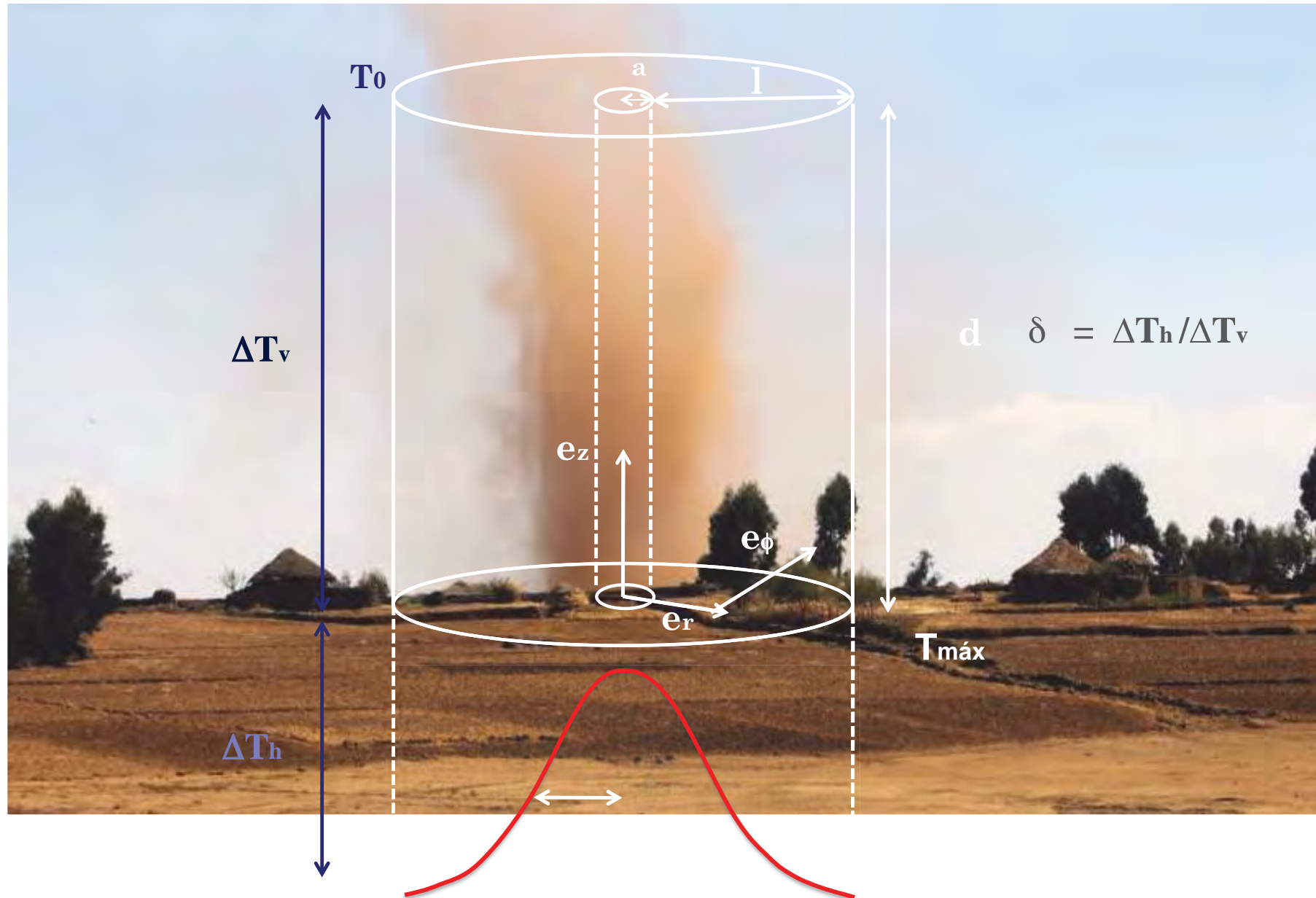
- Localized heating in a cylindrical domain,
- we look for mass transport,
- approximation to atmospheric phenomena

## **Dust devil**

# Outline

- Formulation
- Numerical methods
- Numerical results
- Comparison with observations

# Domain



## Hidrodynamic equations + Heat equation

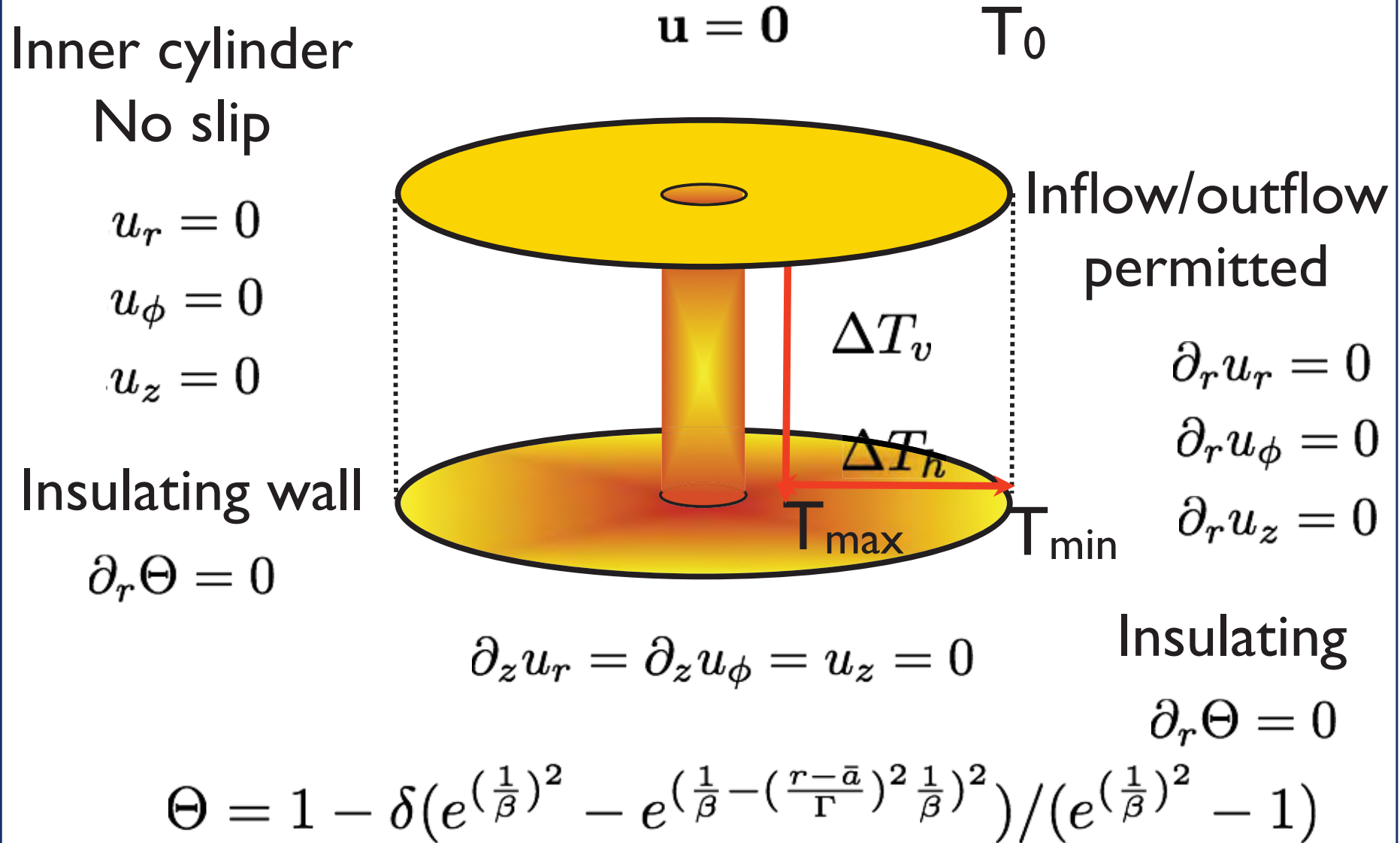
$$\nabla \cdot \mathbf{u} = 0$$

$$\partial_t \Theta + \mathbf{u} \cdot \nabla \Theta = \nabla^2 \Theta$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = Pr (-\nabla p + \nabla^2 \mathbf{u} + R\Theta \mathbf{e}_z)$$

0.7 air

# Cylindrical annulus





## **Time dependent problem**

- Combination of Adams-Bashforth and backward differentiation formula (AB/BDF) scheme for time evolution (second order)
- Fourier and Chebyshev collocation
- Predictor-corrector scheme
  - Predictor Poisson for pressure
  - Predictor Helmholtz for velocity
  - Corrector

### 3. Time dependent problem

The screenshot shows the 'iMac' System Information window. The left sidebar lists hardware categories, with 'Hardware' expanded. The main pane shows 'Información del hardware:' with the following details:

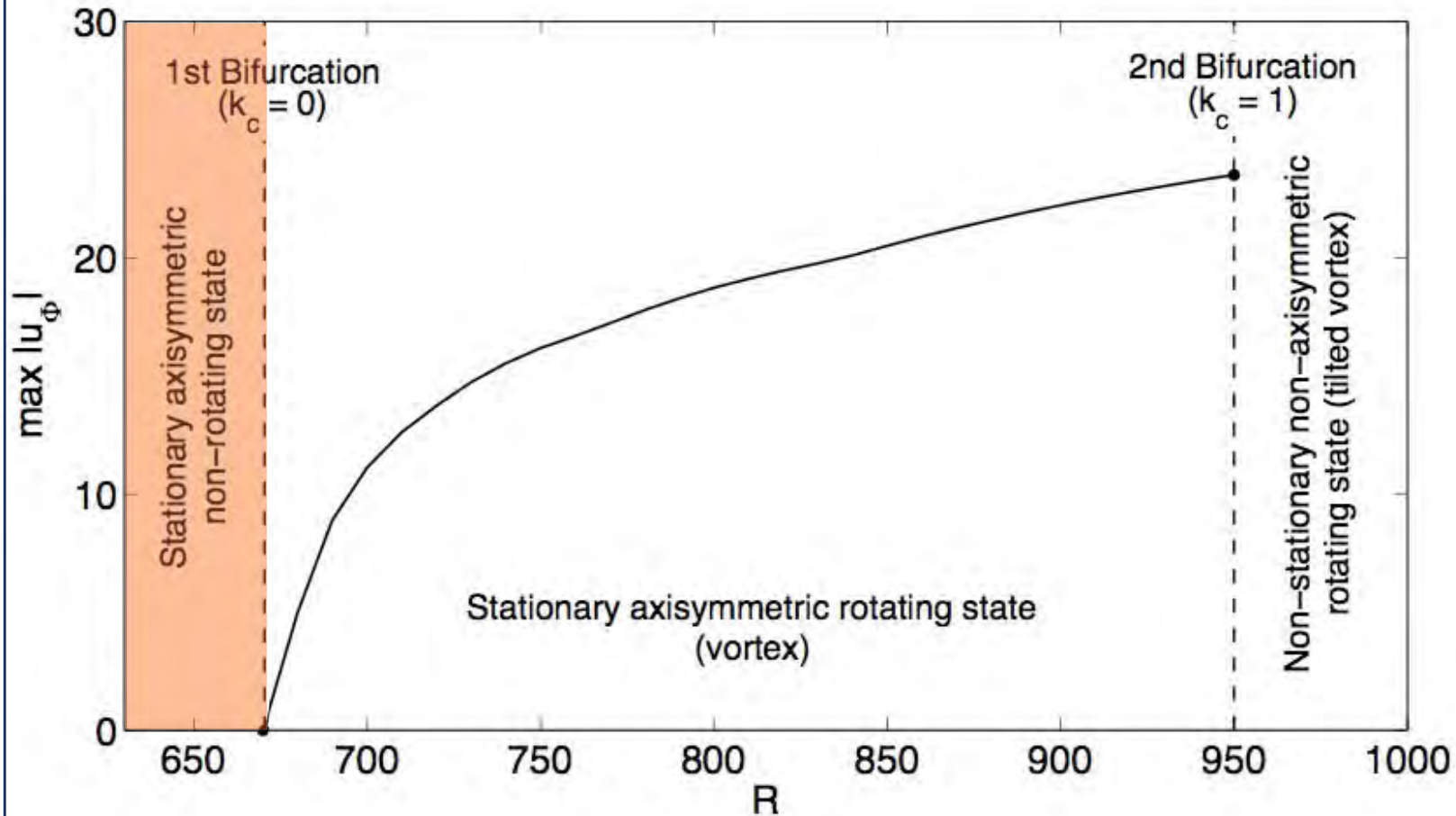
Nombre del modelo:	iMac
Identificador del modelo:	iMac17,1
Nombre del procesador:	Intel Core i7
Velocidad del procesador:	4 GHz
Cantidad de procesadores:	1
Cantidad total de núcleos:	4
Caché de nivel 2 (por núcleo):	256 KB
Caché de nivel 3:	8 MB
Memoria:	32 GB

Below this, there are tabs for 'Visión general', 'Pantallas', 'Almacenamiento', 'Memoria', 'Soporte', and 'Servicio'. The 'Visión general' tab is active, showing the OS X El Capitan logo and version 10.11.4. Below the logo, the system model is listed as 'iMac (Retina 5K, 27 pulgadas, finales de 2015)'. Further details include: 'Procesador 4 GHz Intel Core i7', 'Memoria 32 GB 1867 MHz DDR3', and 'Gráficos AMD Radeon R9 M390 2048 MB'. At the bottom of the window, the text 'iMac de Damián > Hardware' is visible.

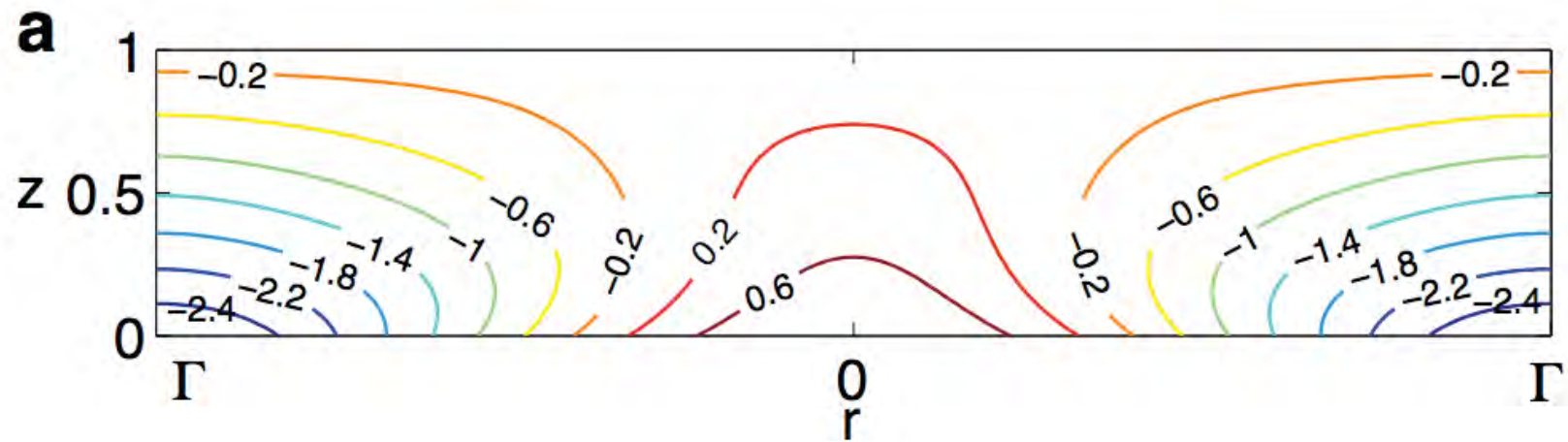
$dt = 0.0001$ ,  
50.000 spatial  
nodes, 0.8 s

# Plume

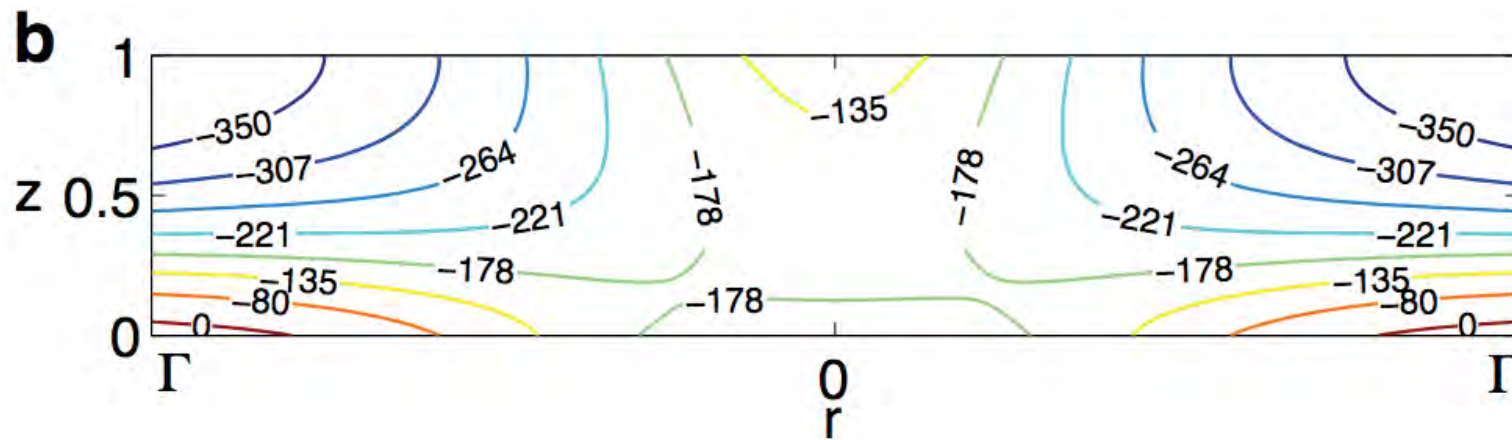
## Bifurcation diagram



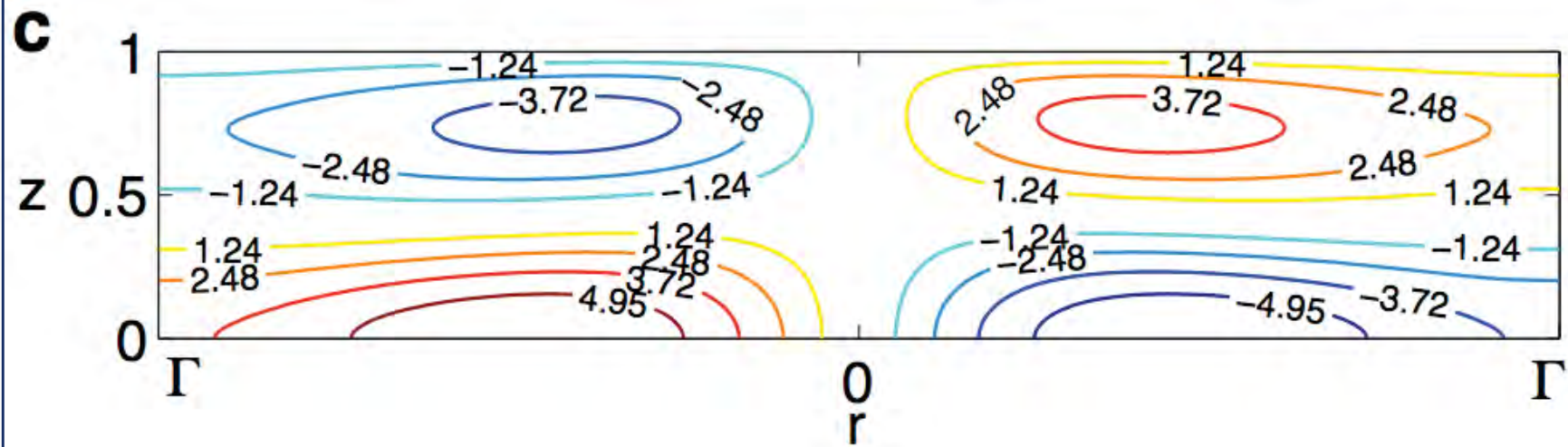
# Temperature



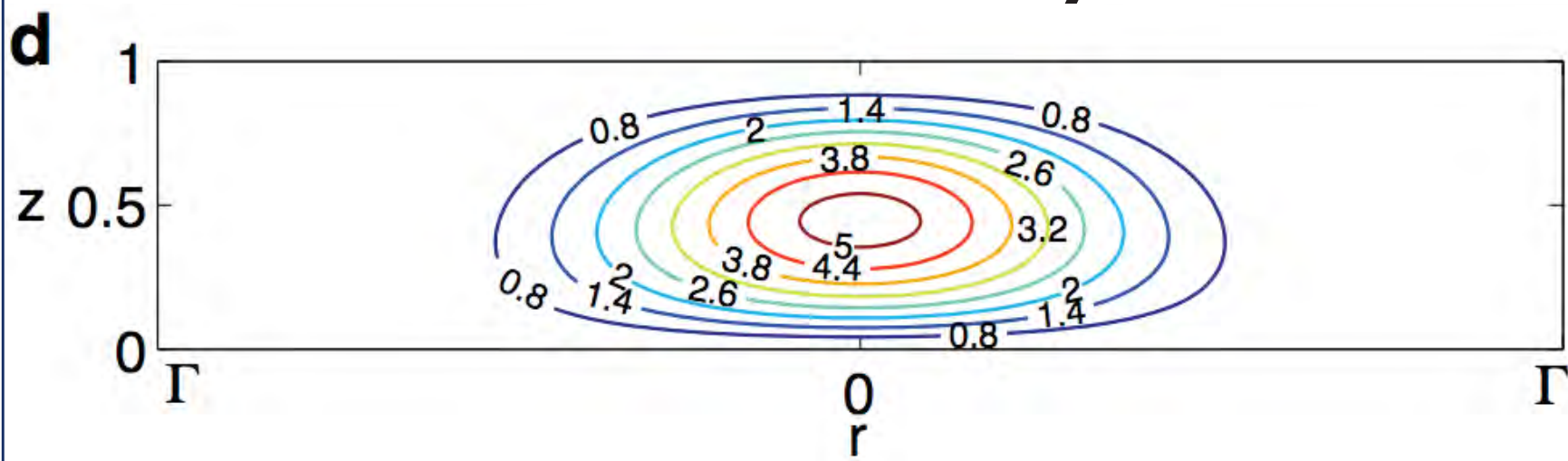
# Pressure



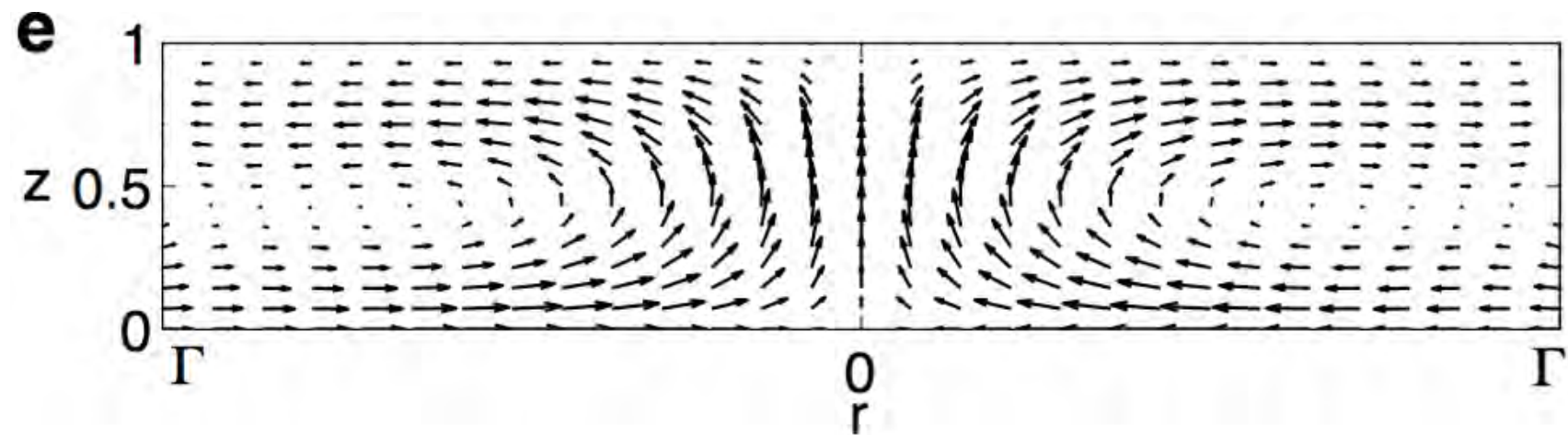
# Radial velocity



# Vertical velocity

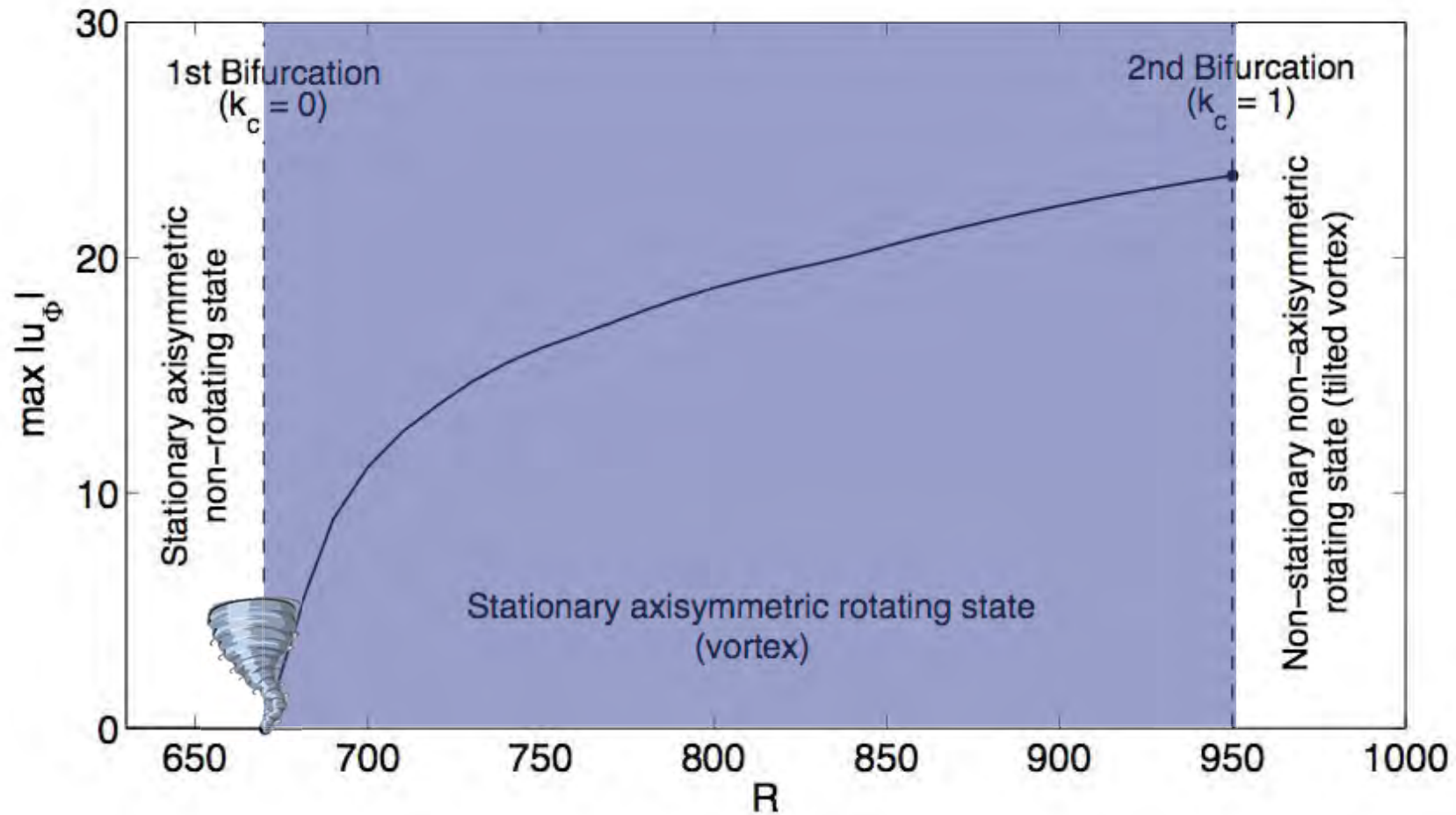


## Velocity field



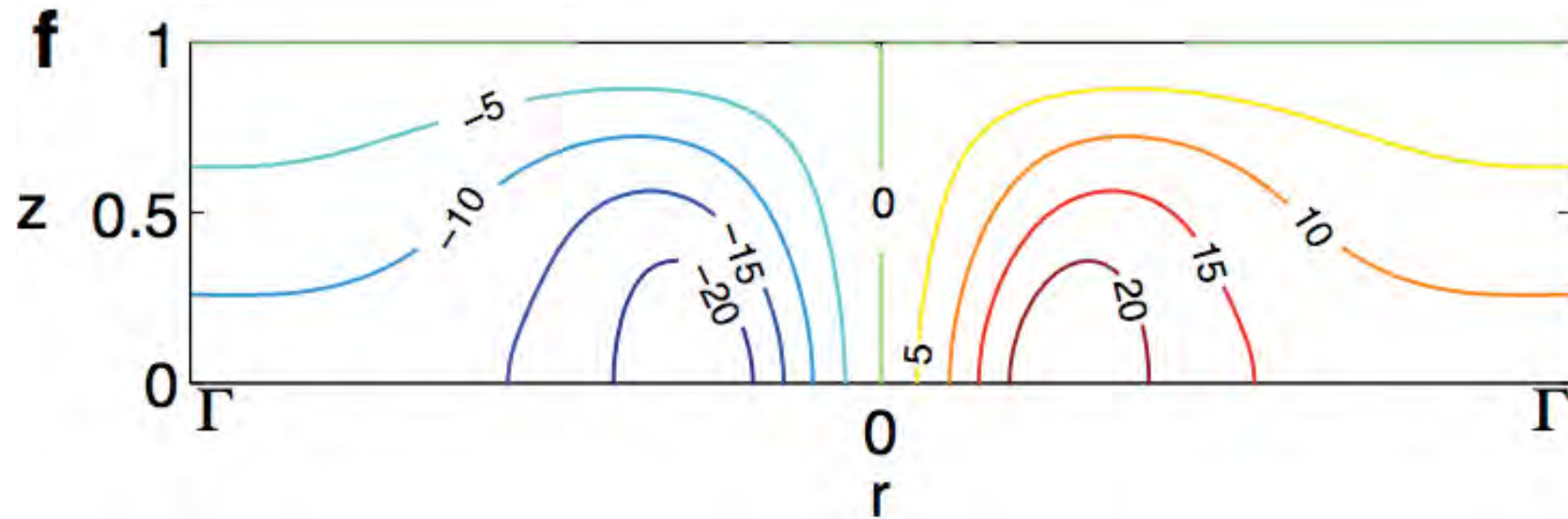
**NO azimuthal velocity**

# Bifurcation diagram



Vertical vortex

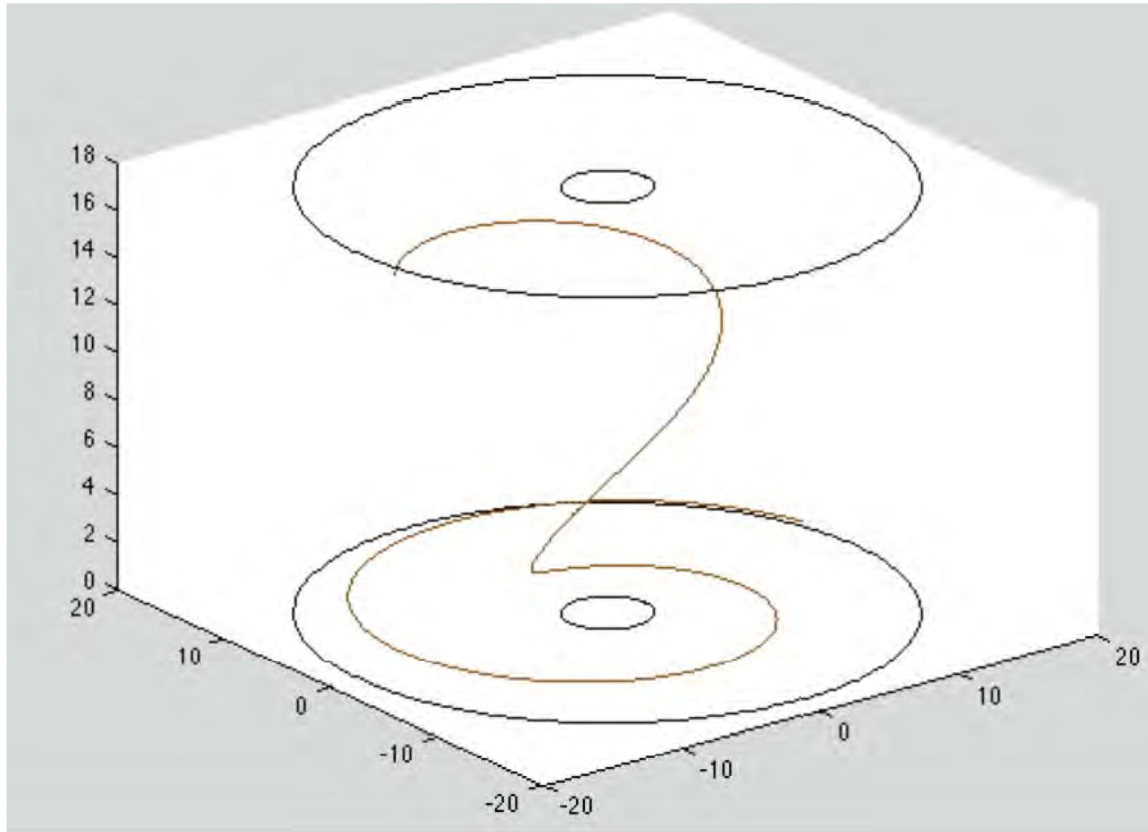
# Azimuthal velocity



**Spin motion**



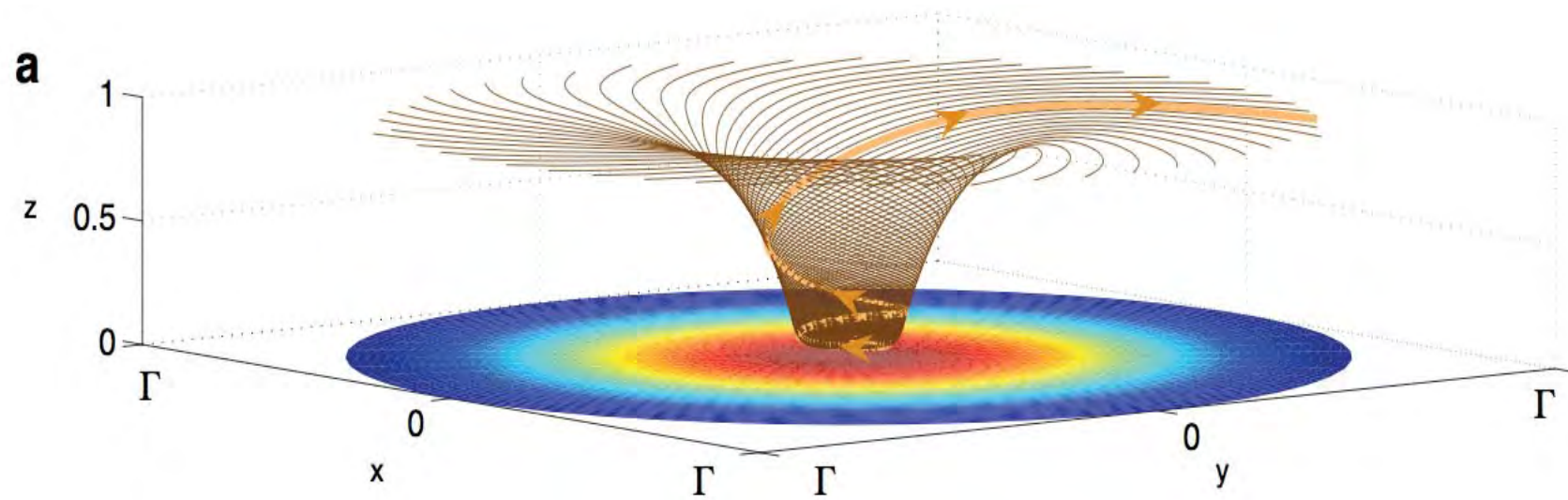
# Trajectory of particles



$$\frac{\partial r}{\partial t} = u_r(r, z)$$
$$\frac{\partial \phi}{\partial t} = u_\phi(r, z)$$
$$\frac{\partial z}{\partial t} = u_z(r, z)$$

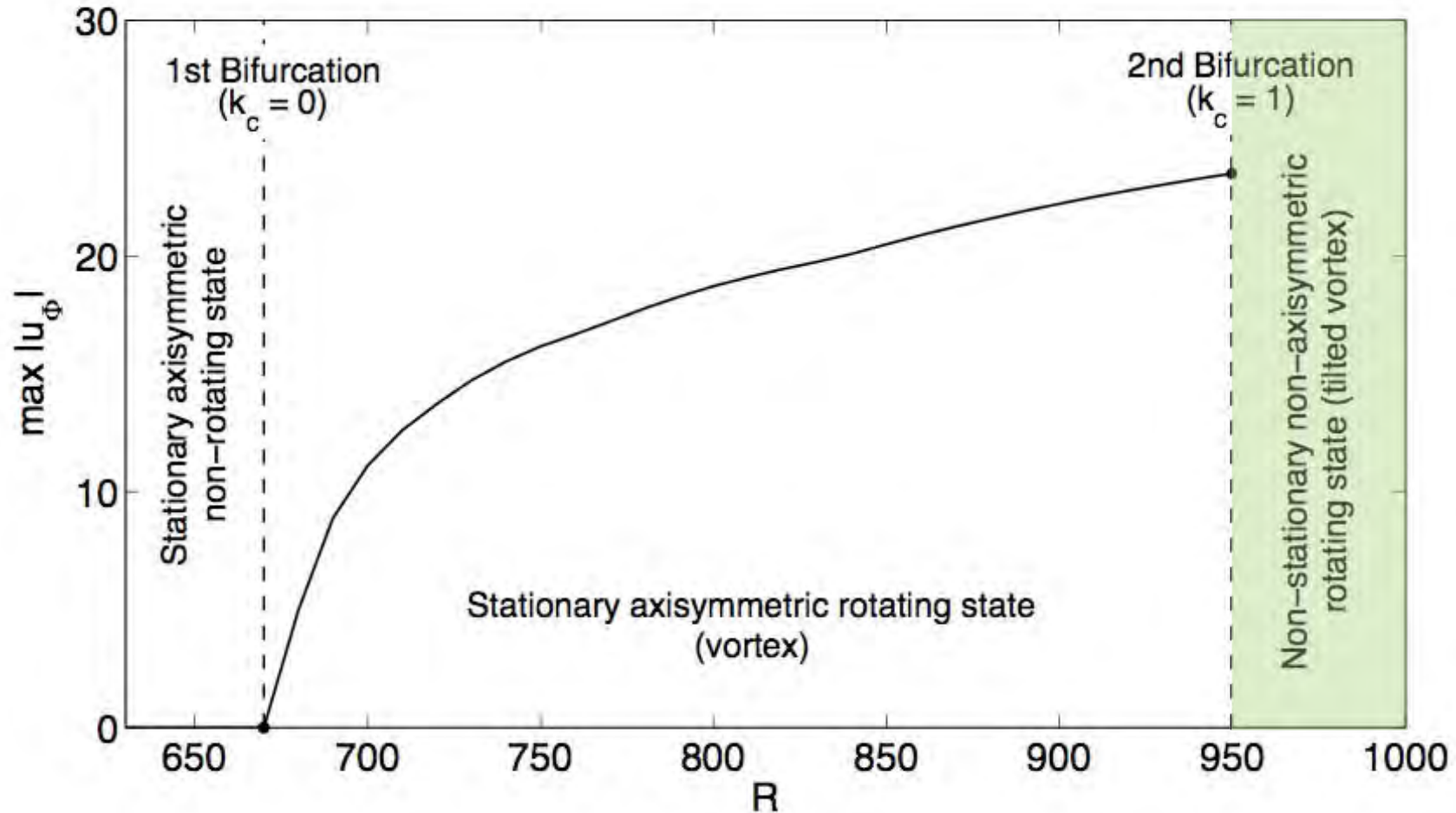
Integrating the velocity fields with respect to time

# Trajectory of particles



Oscillatory

# Bifurcation diagram



# Bifurcation diagram scheme

FP → FP → **Limit cycle**

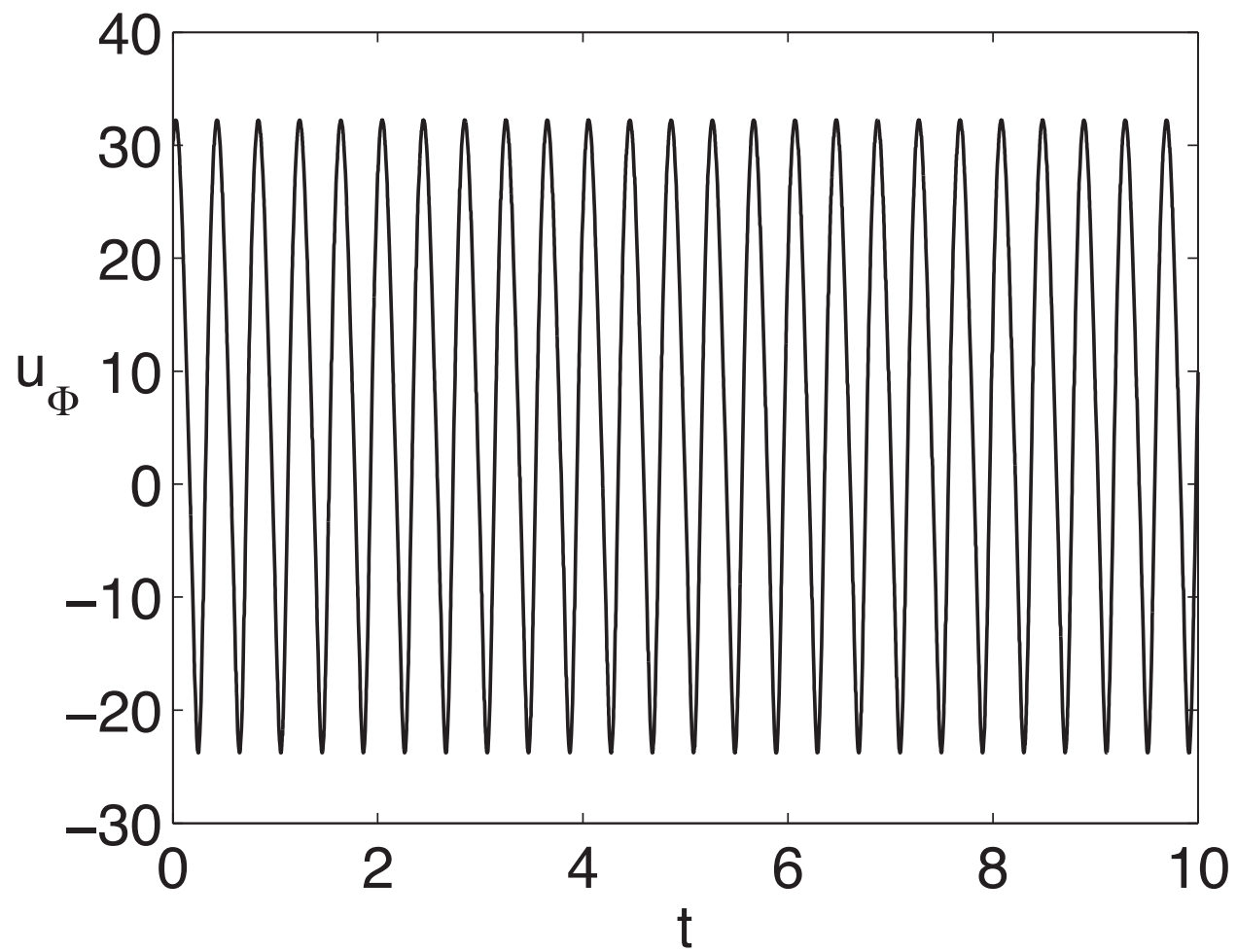
R



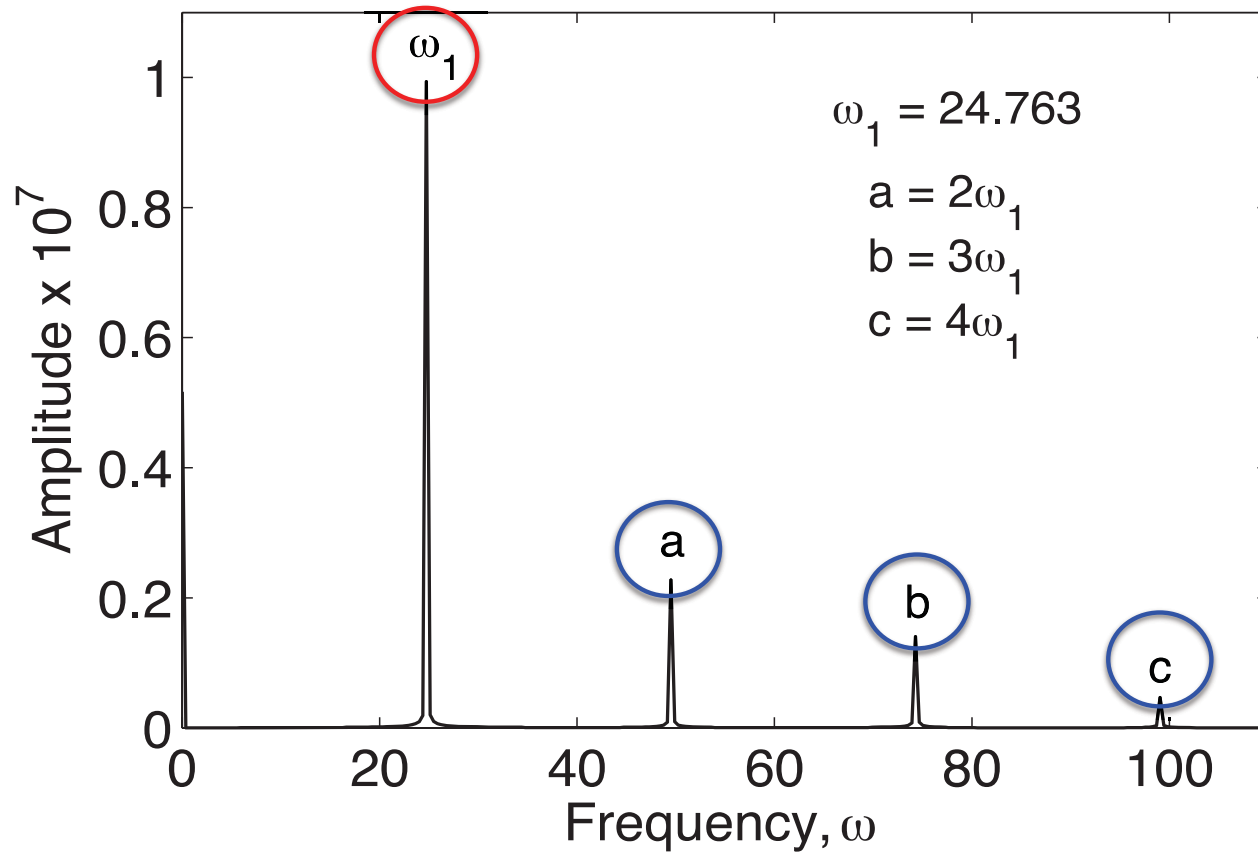
472

**3527**

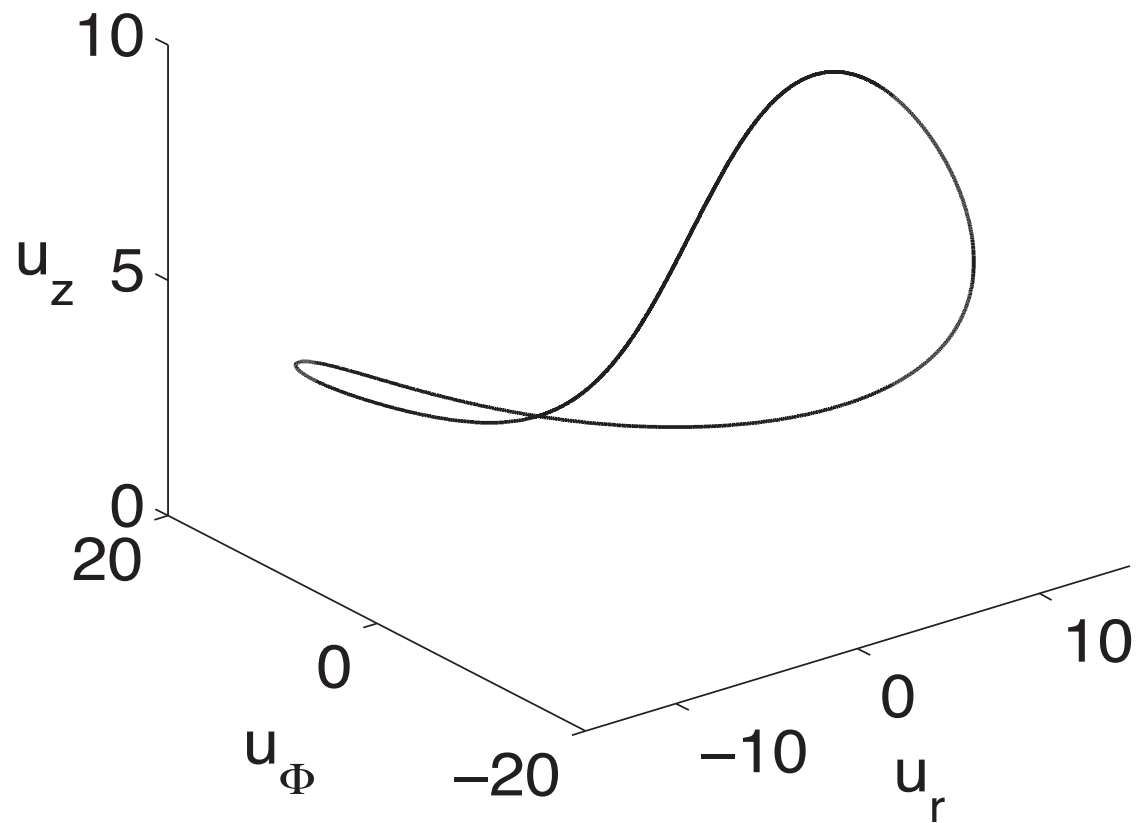
$Ra = 10000$

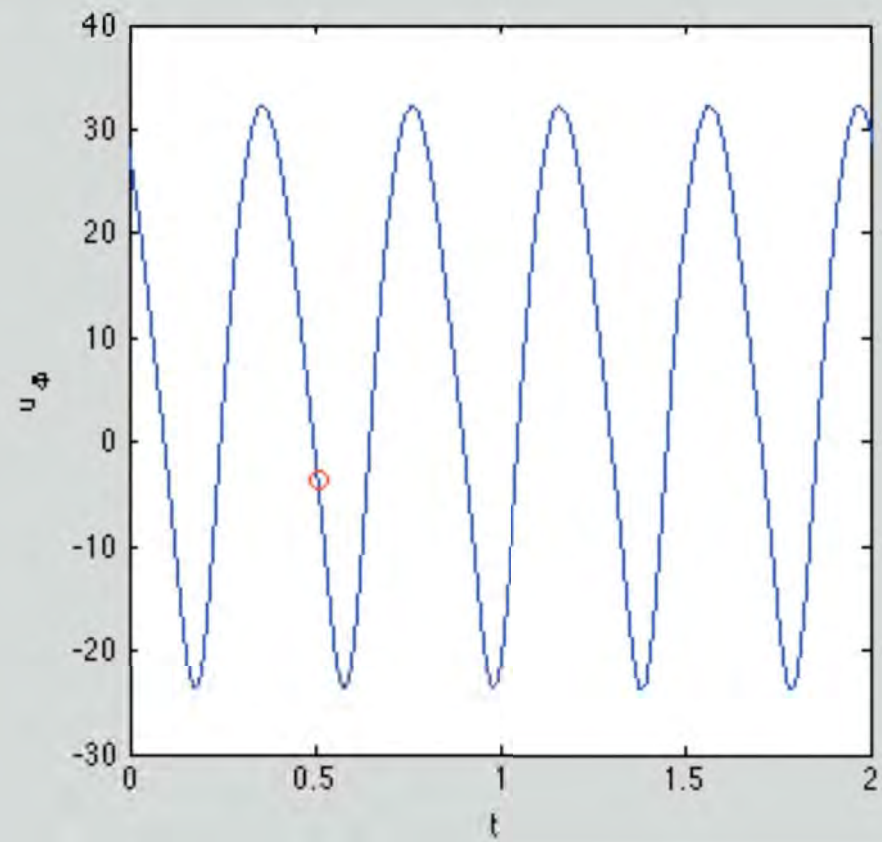
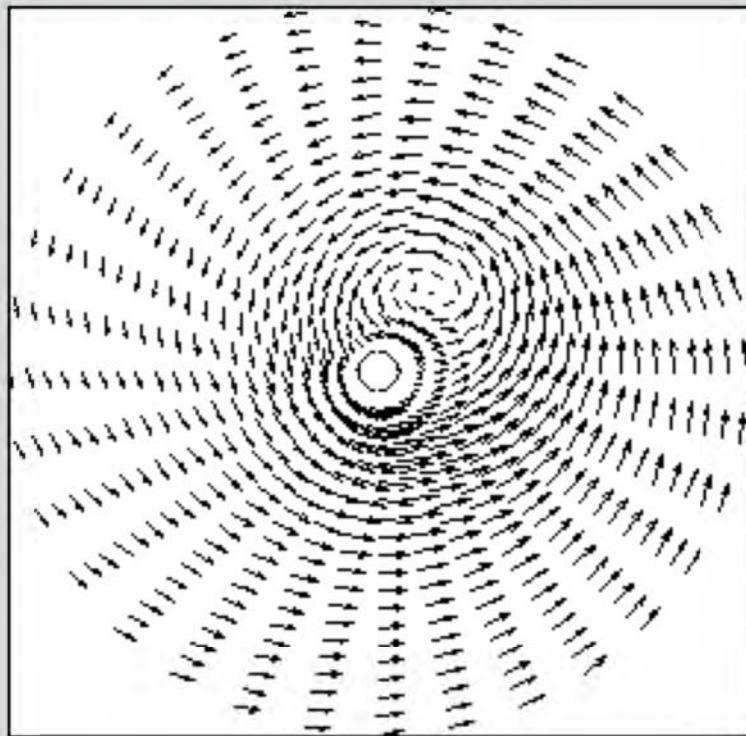


$Ra = 10000$



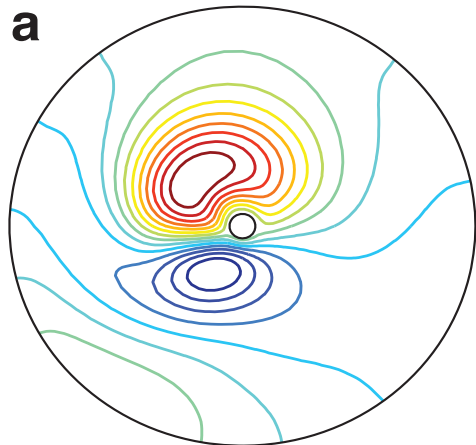
$Ra = 10000$



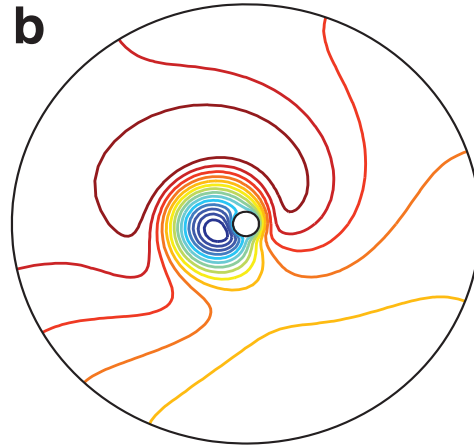




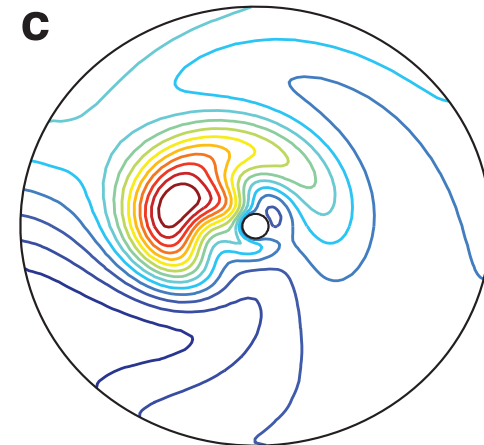
$Ra = 10000$



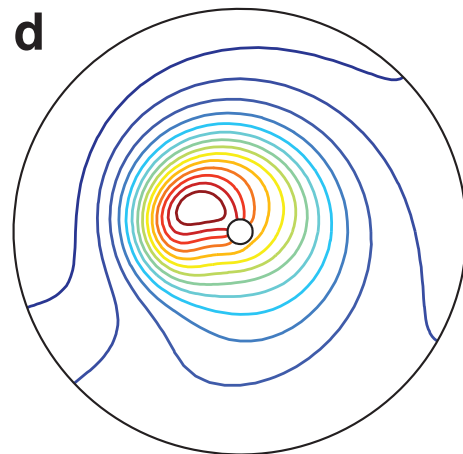
$u_r$



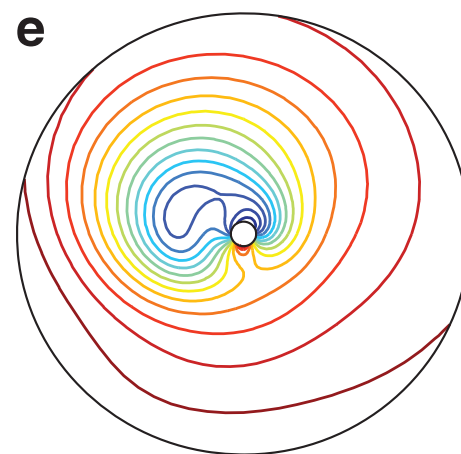
$u_\phi$



$u_z$



$t$



$p$

# Bifurcation diagram scheme

FP → FP → LC → T<sup>2</sup> Torus

R

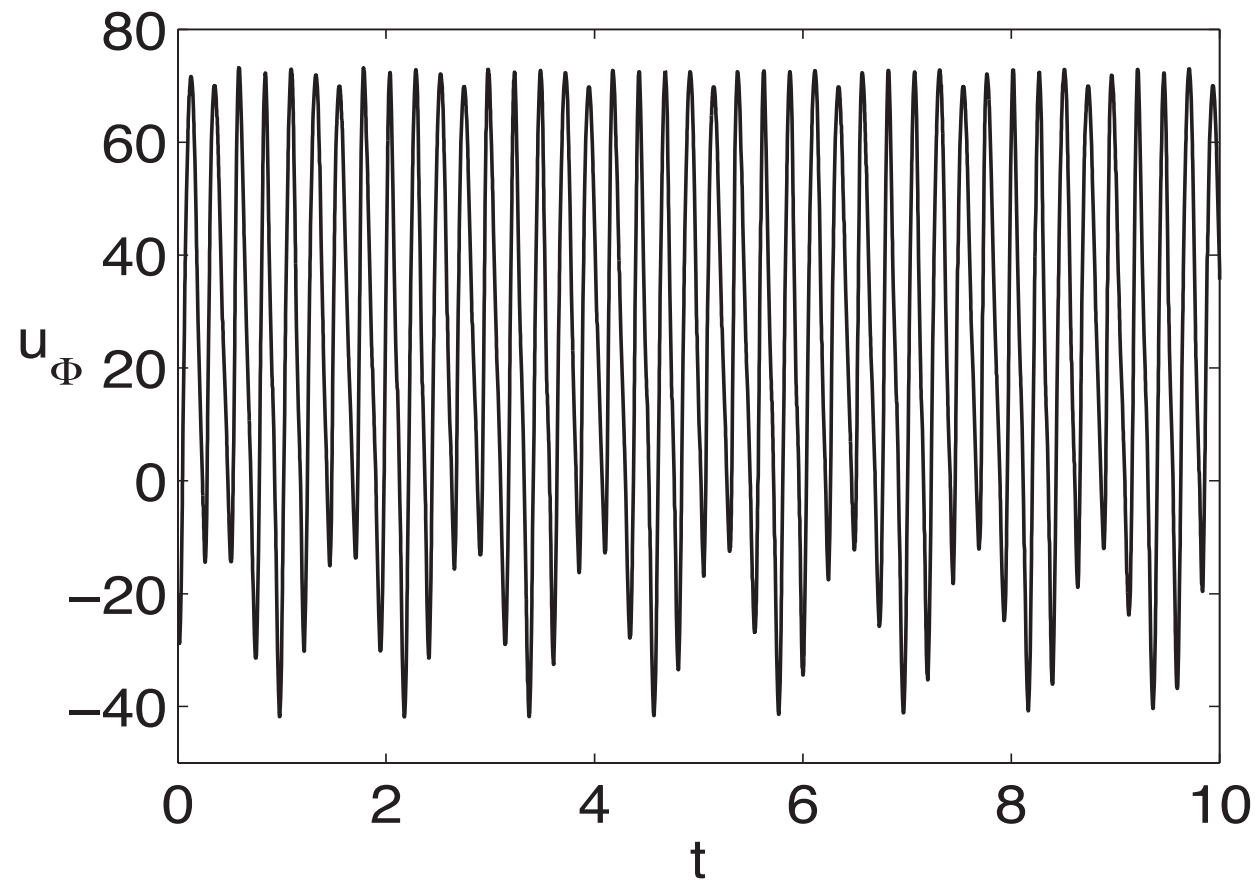


472

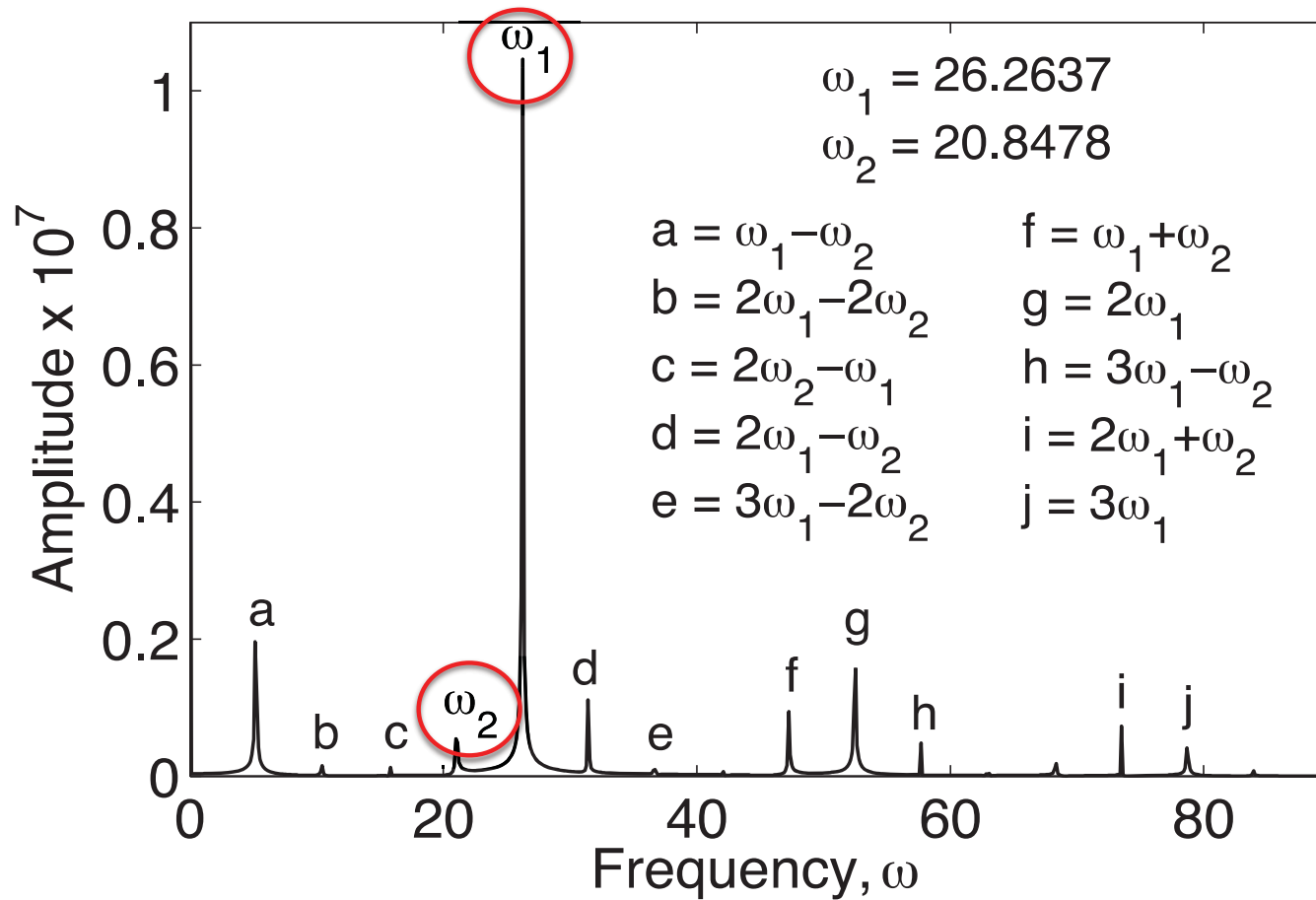
3527

**25000**

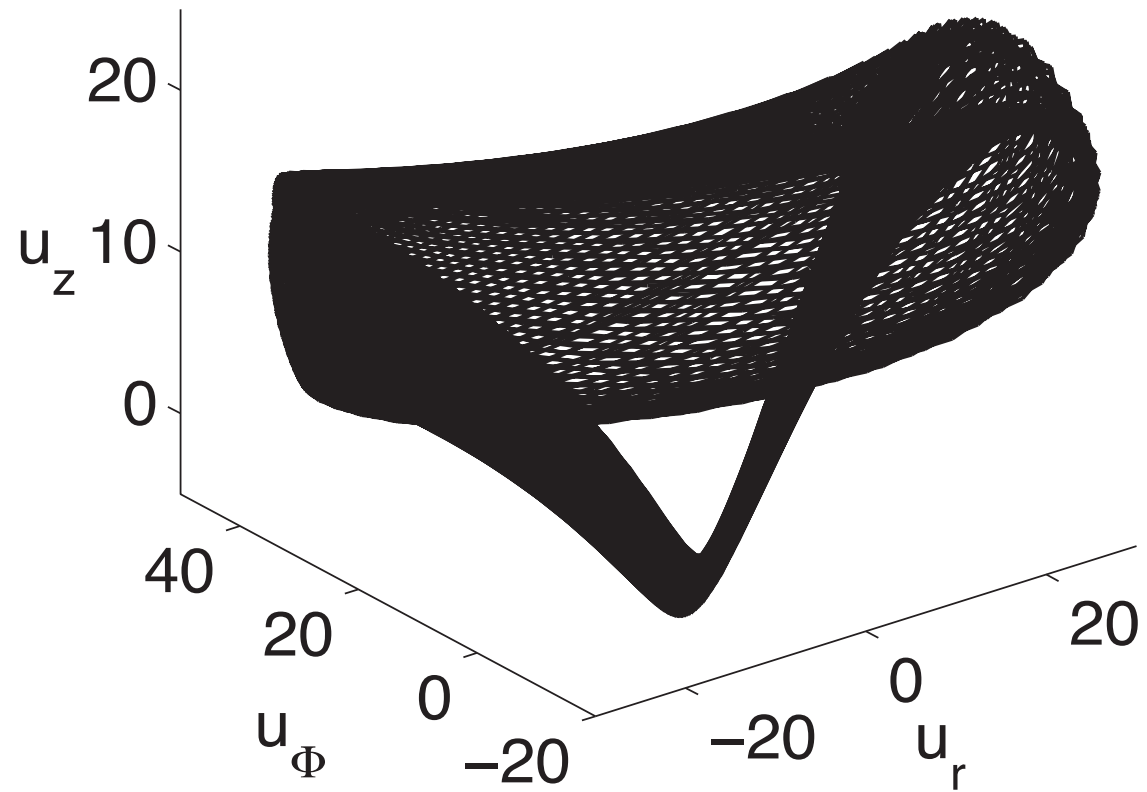
Ra = 35000

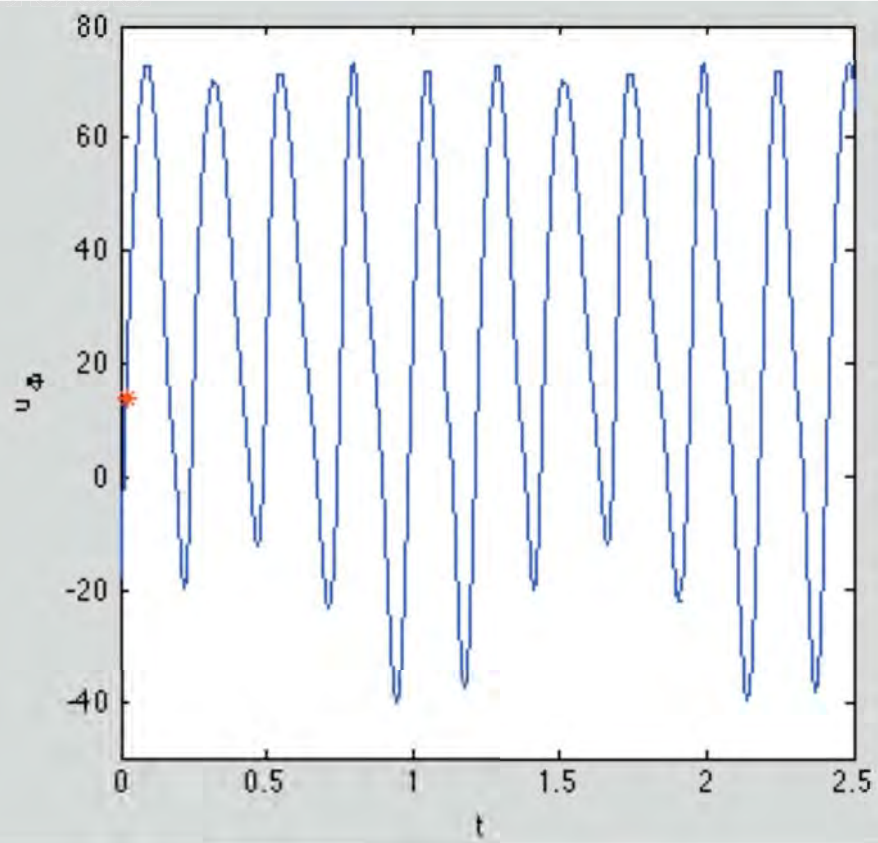
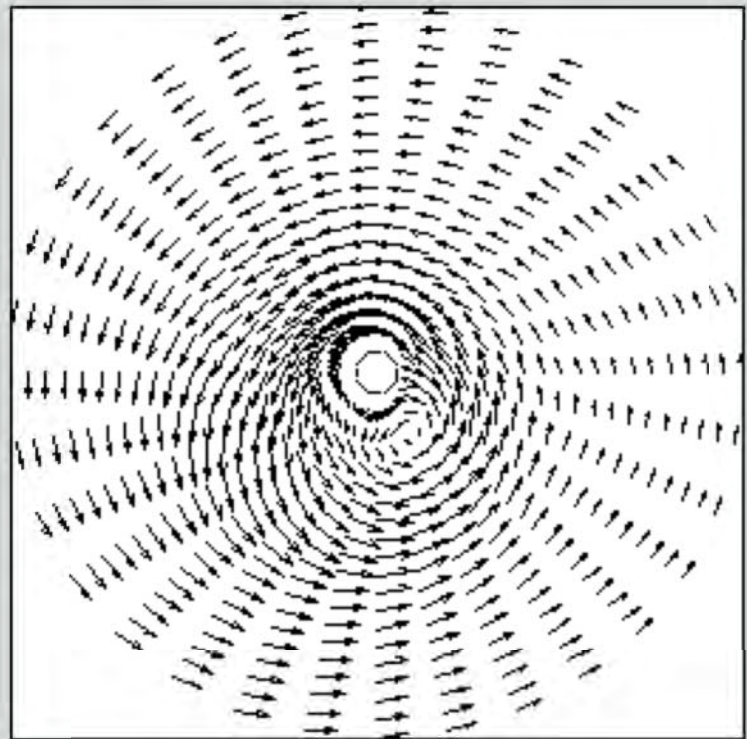


Ra = 35000



$Ra = 35000$





# Bifurcation diagram scheme

FP → FP → LC → T<sup>2</sup>T → **Limit cycle**

R



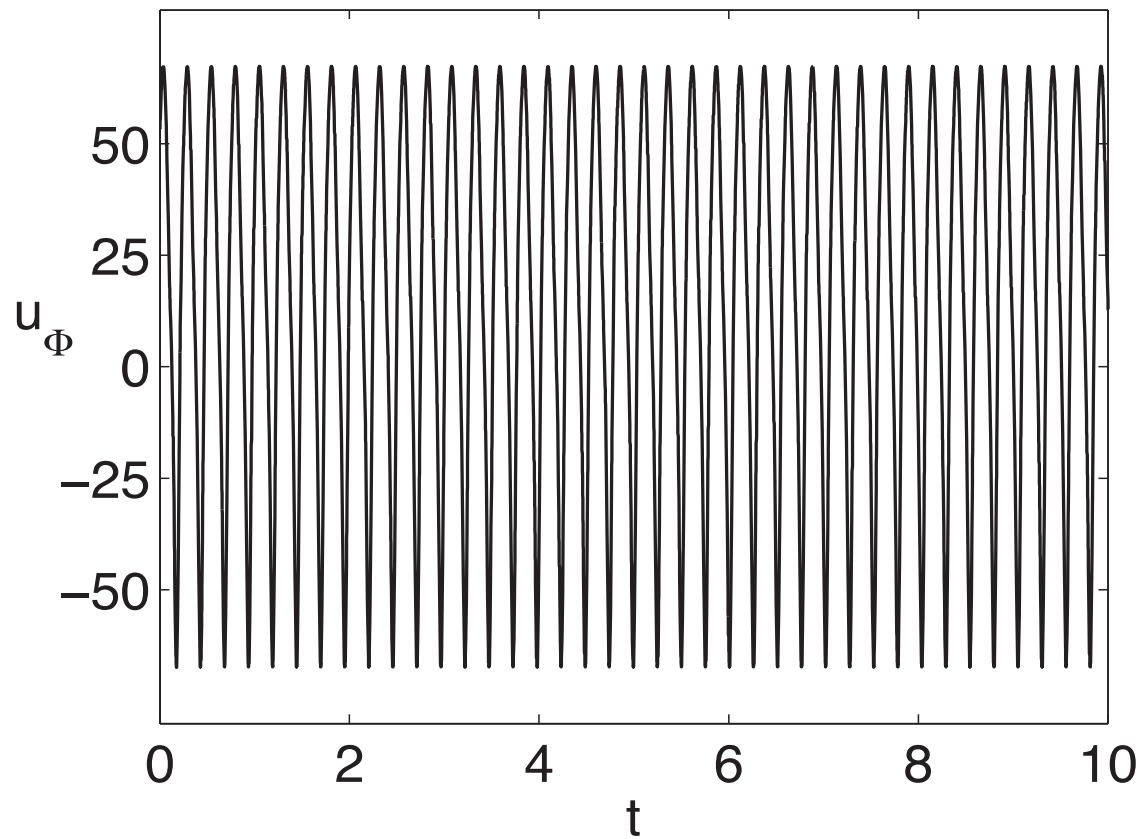
472

3527

25000

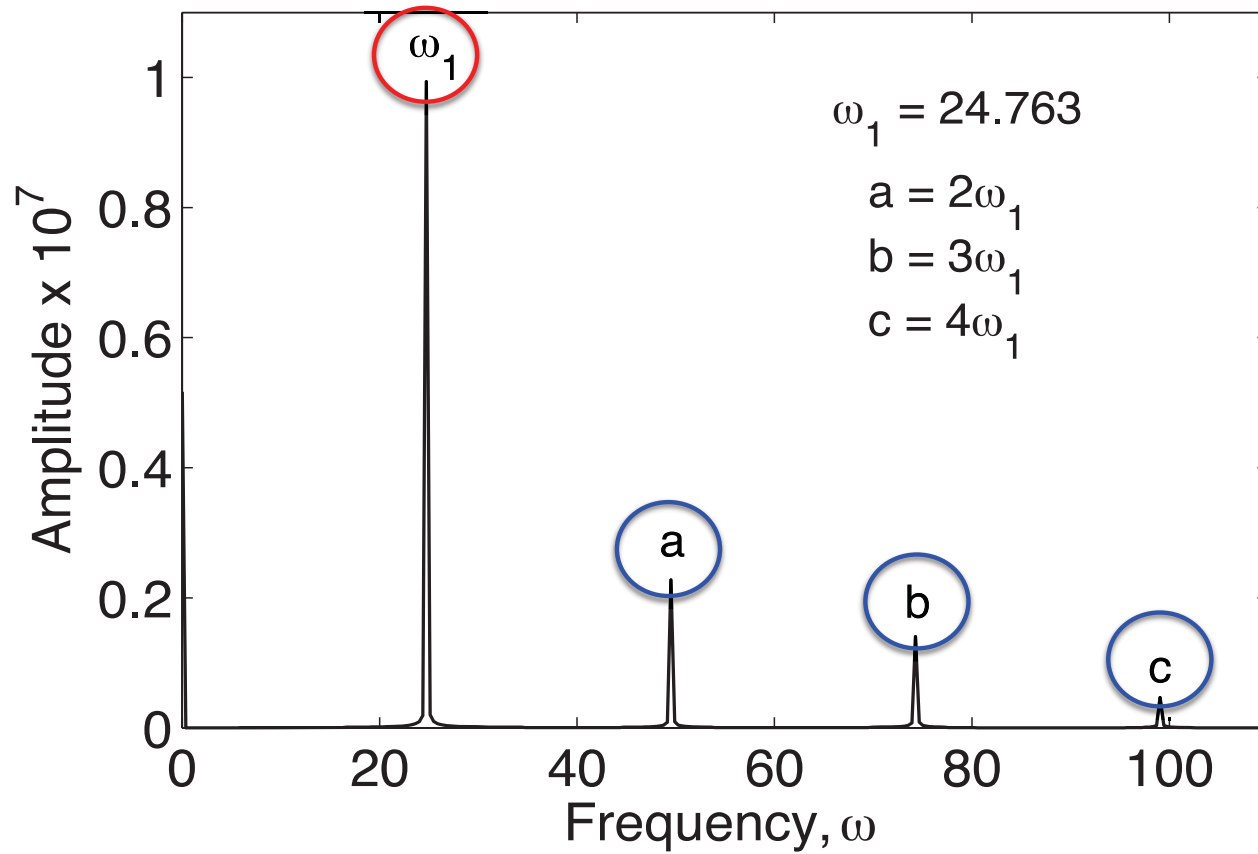
**37500**

$Ra = 40000$

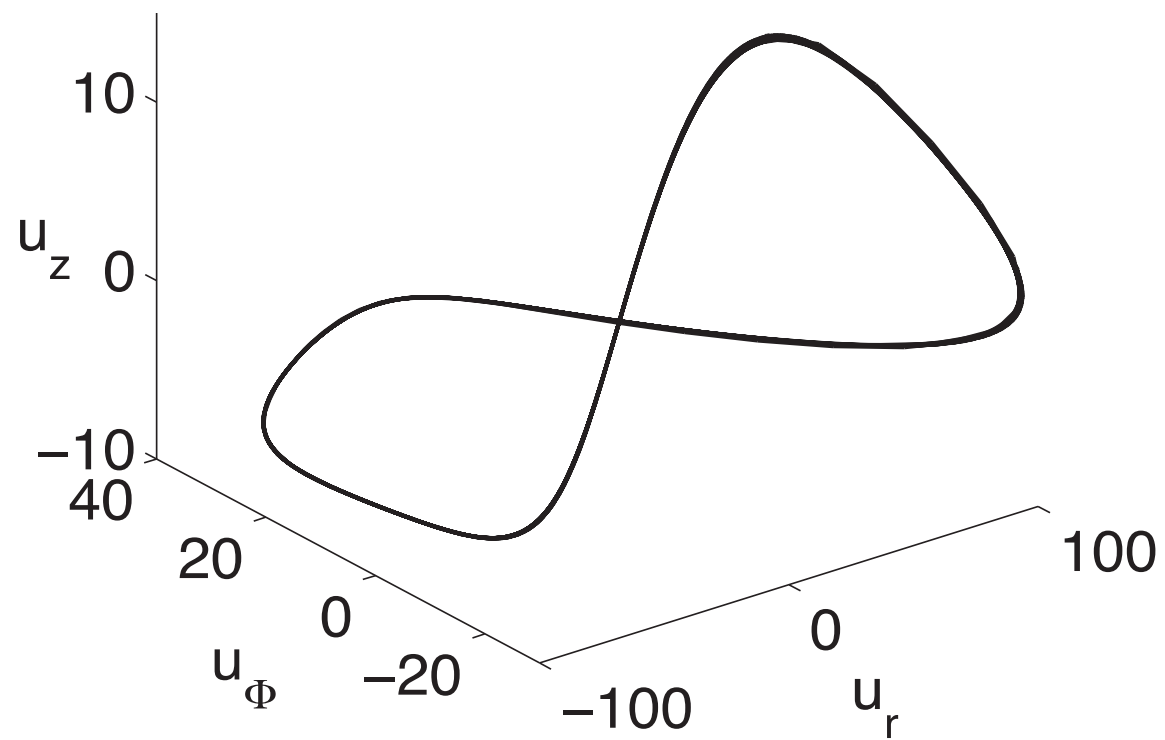




Ra = 40000

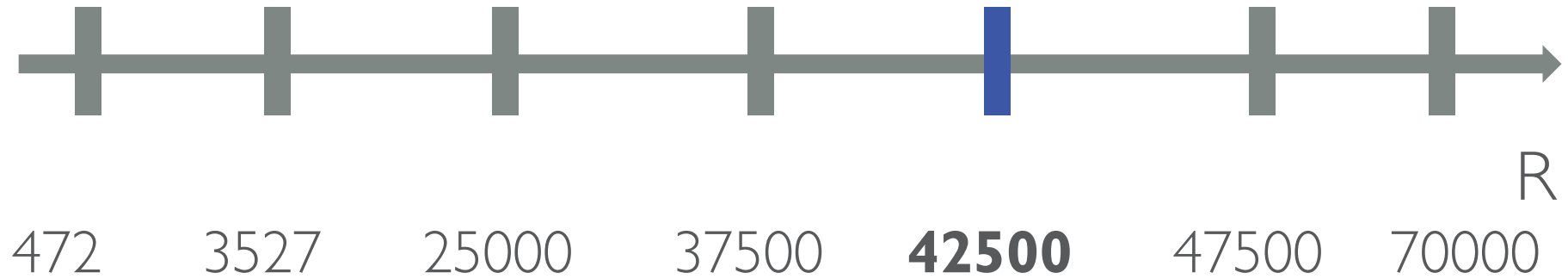


$Ra = 40000$

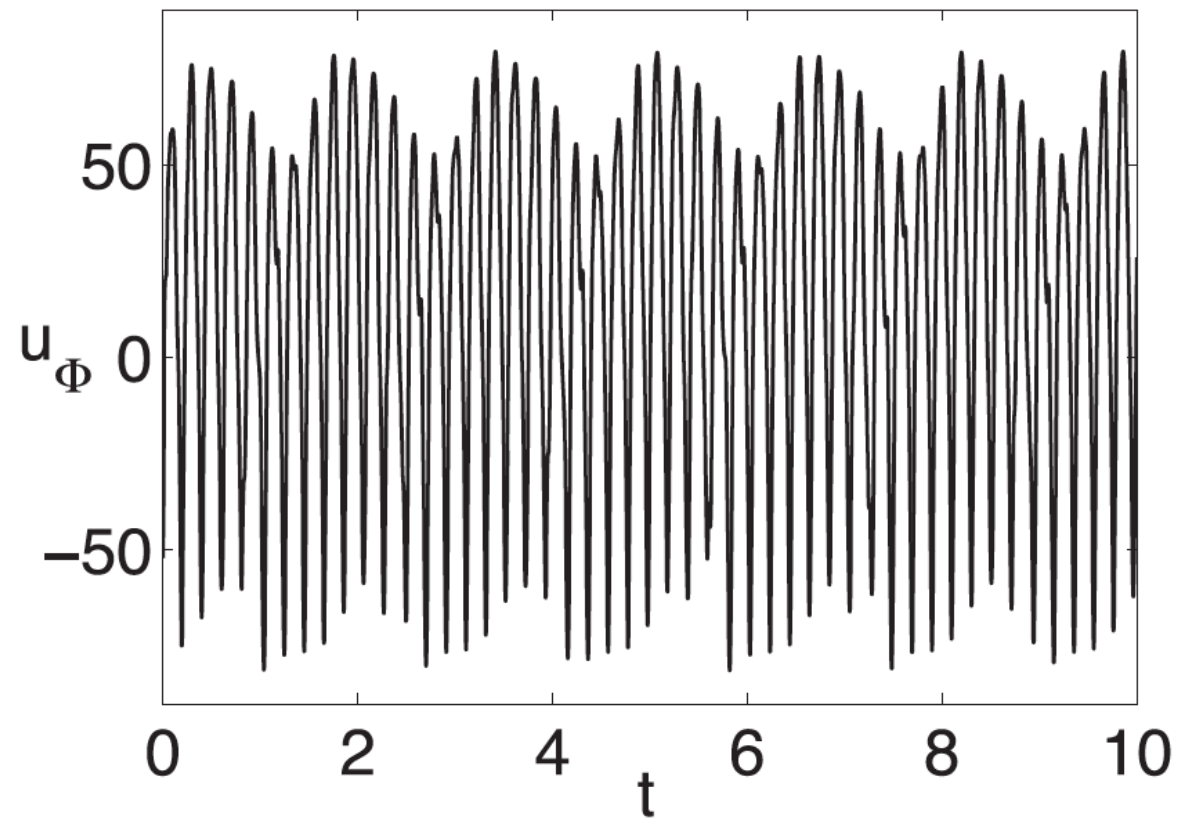


# Bifurcation diagram scheme

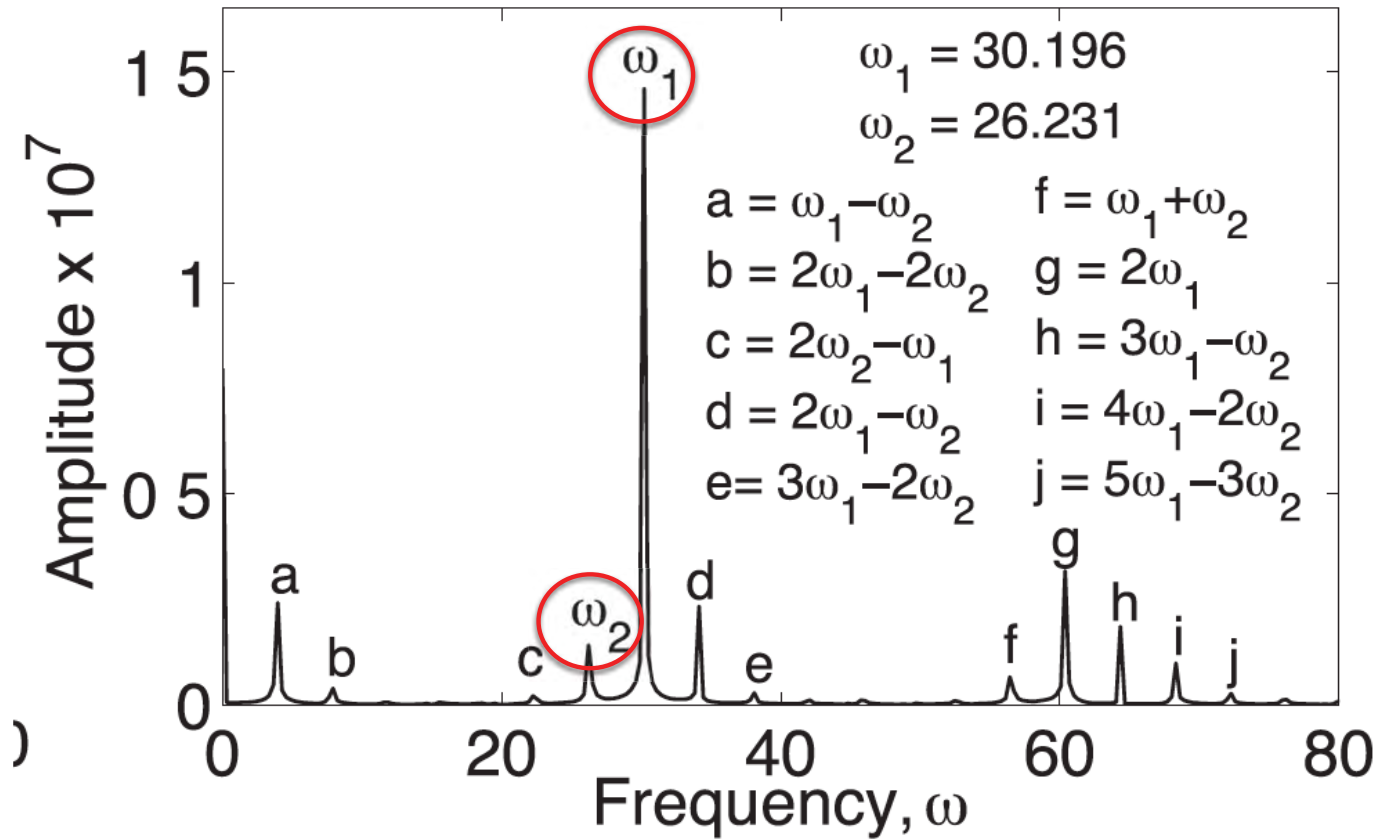
FP → FP → LC → T<sup>2</sup>T → LC → **T<sup>2</sup>Torus**



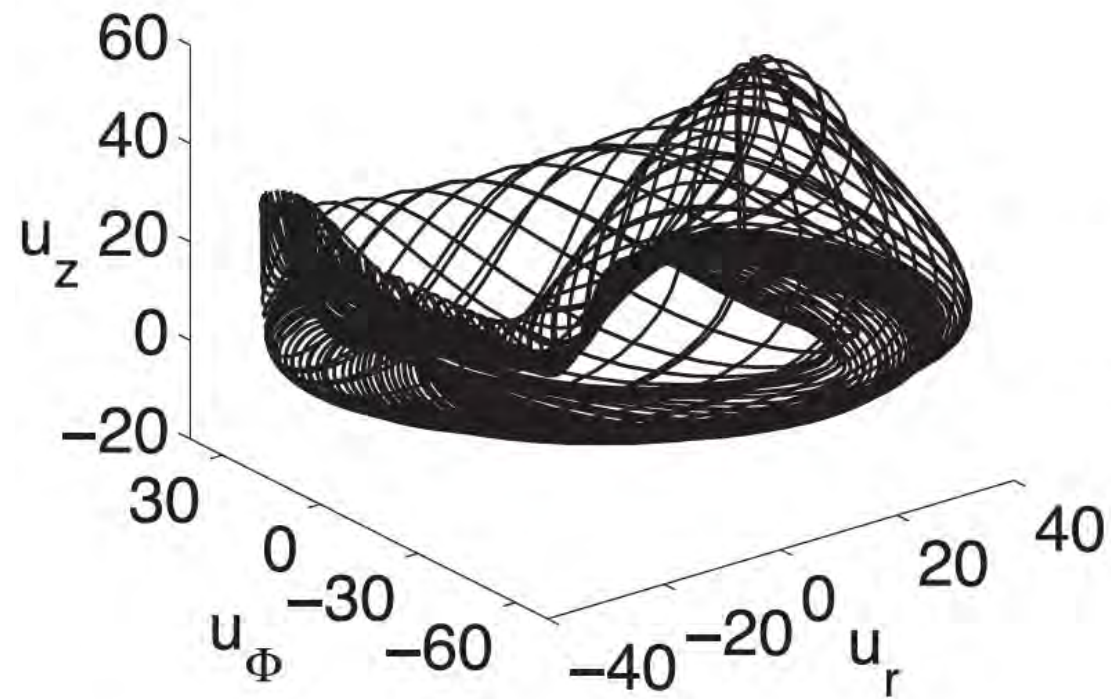
$Ra = 45000$



Ra = 45000

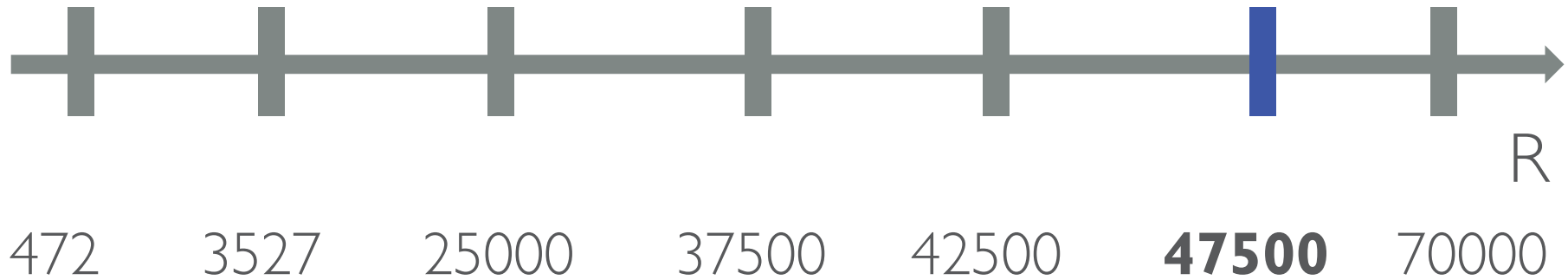


$Ra = 45000$

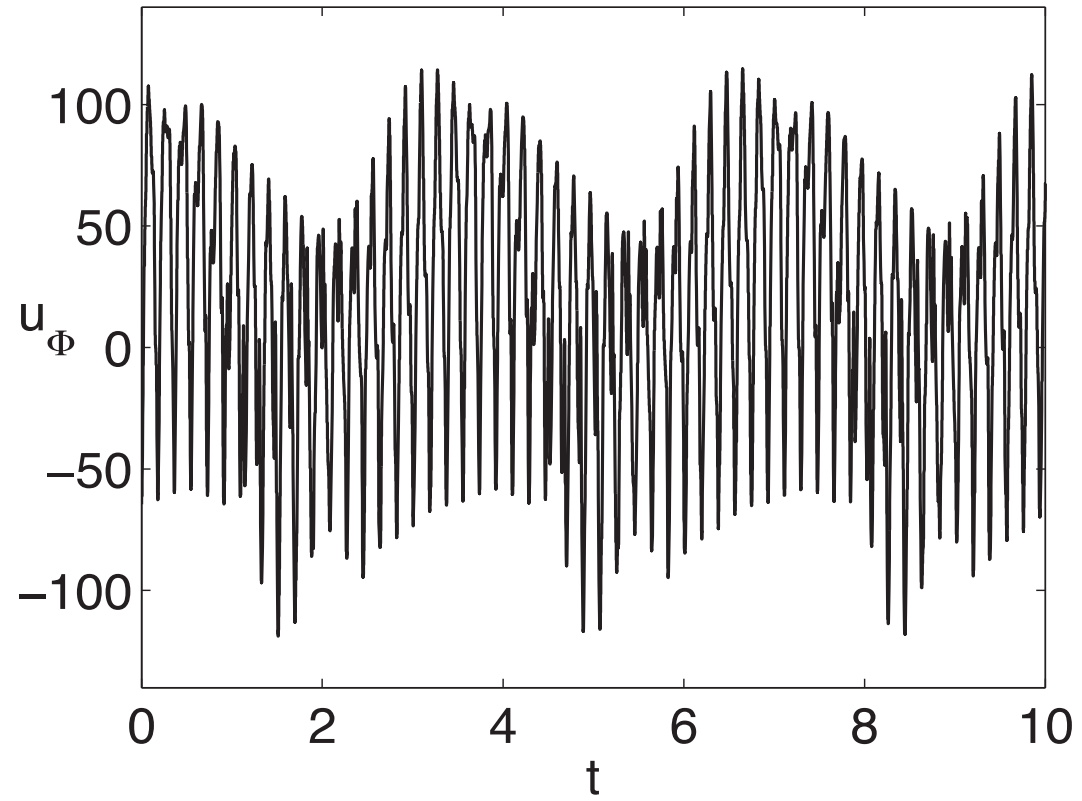


# Bifurcation diagram scheme

FP → FP → LC → T<sup>2</sup>T → LC → T<sup>2</sup>T → **T<sup>3</sup> Torus**

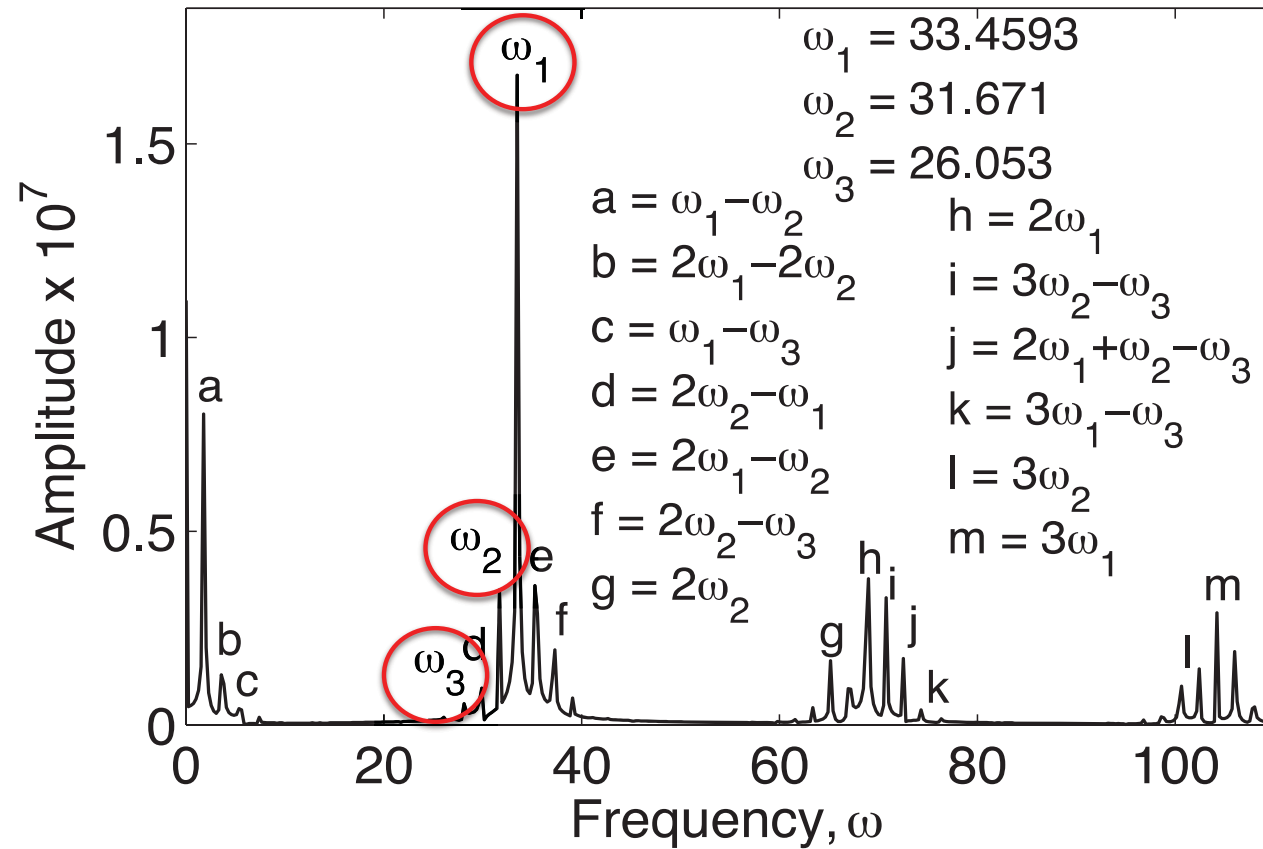


$Ra = 60000$

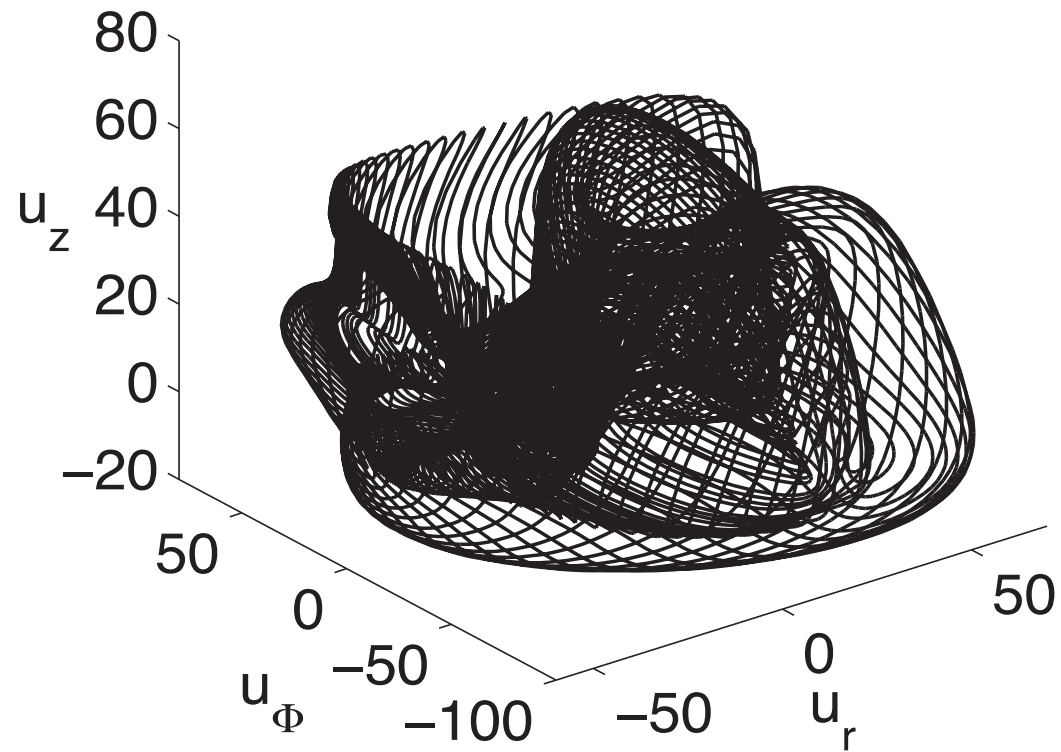


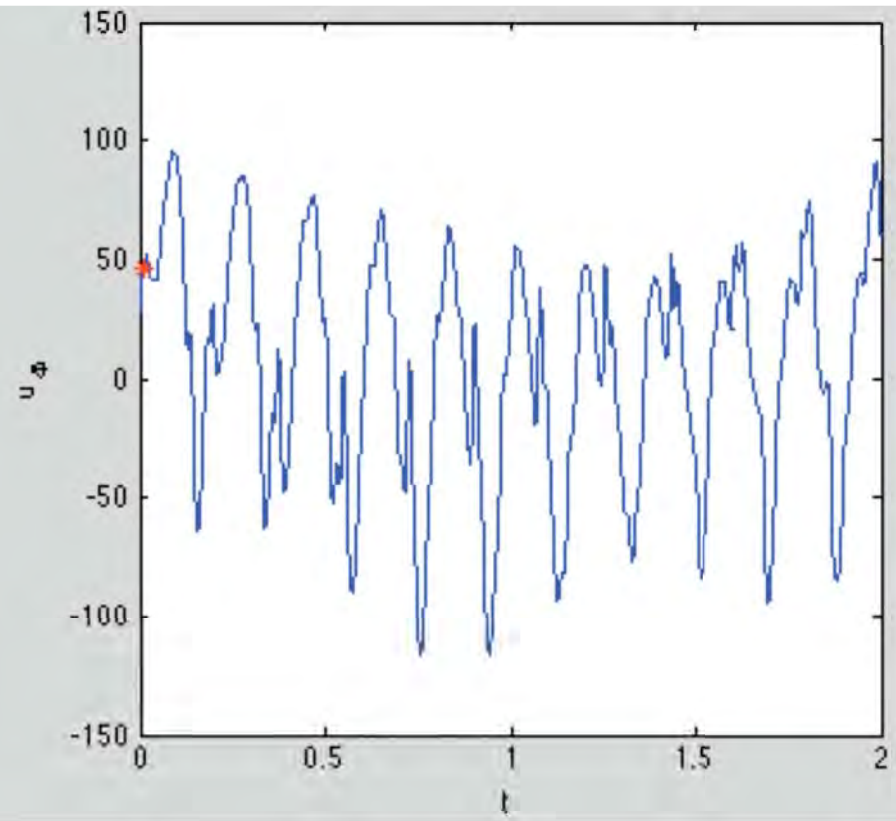
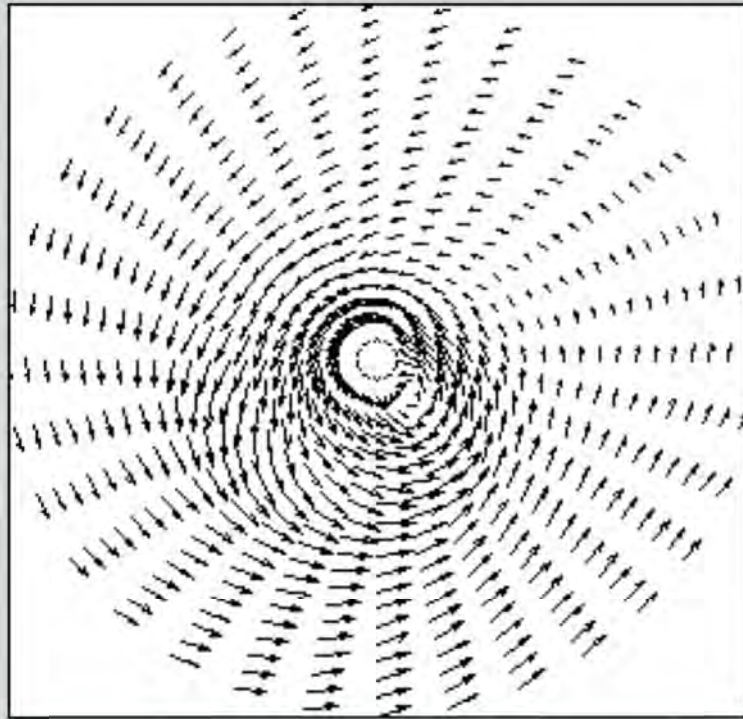


Ra = 60000



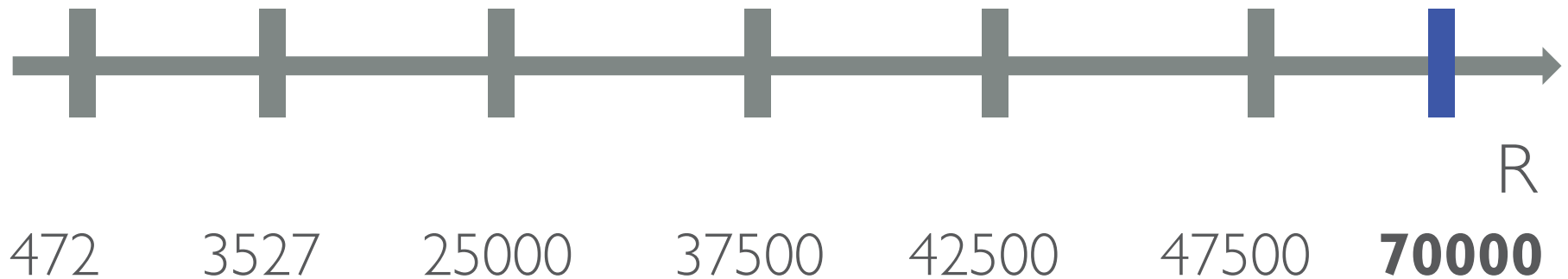
$Ra = 60000$





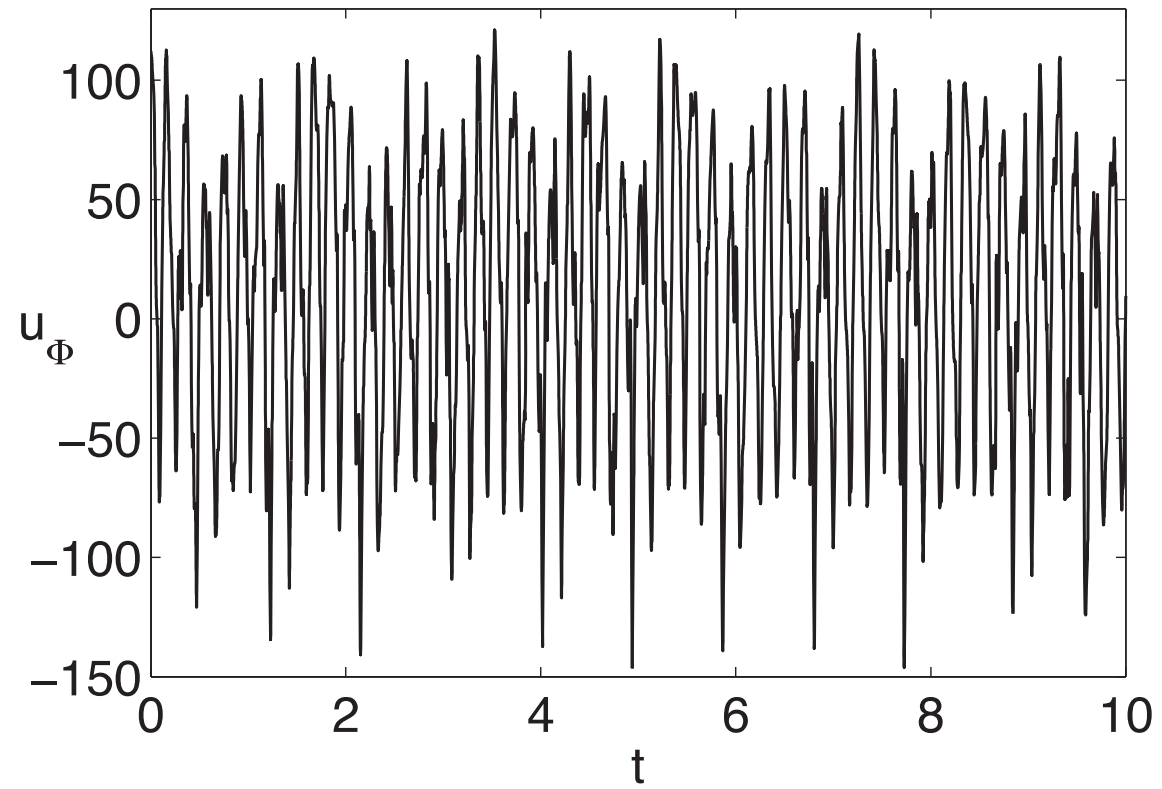
# Bifurcation diagram scheme

FP → FP → LC → T<sup>2</sup>T → LC → T<sup>2</sup>T → T<sup>3</sup>T → **Extrange atractor**

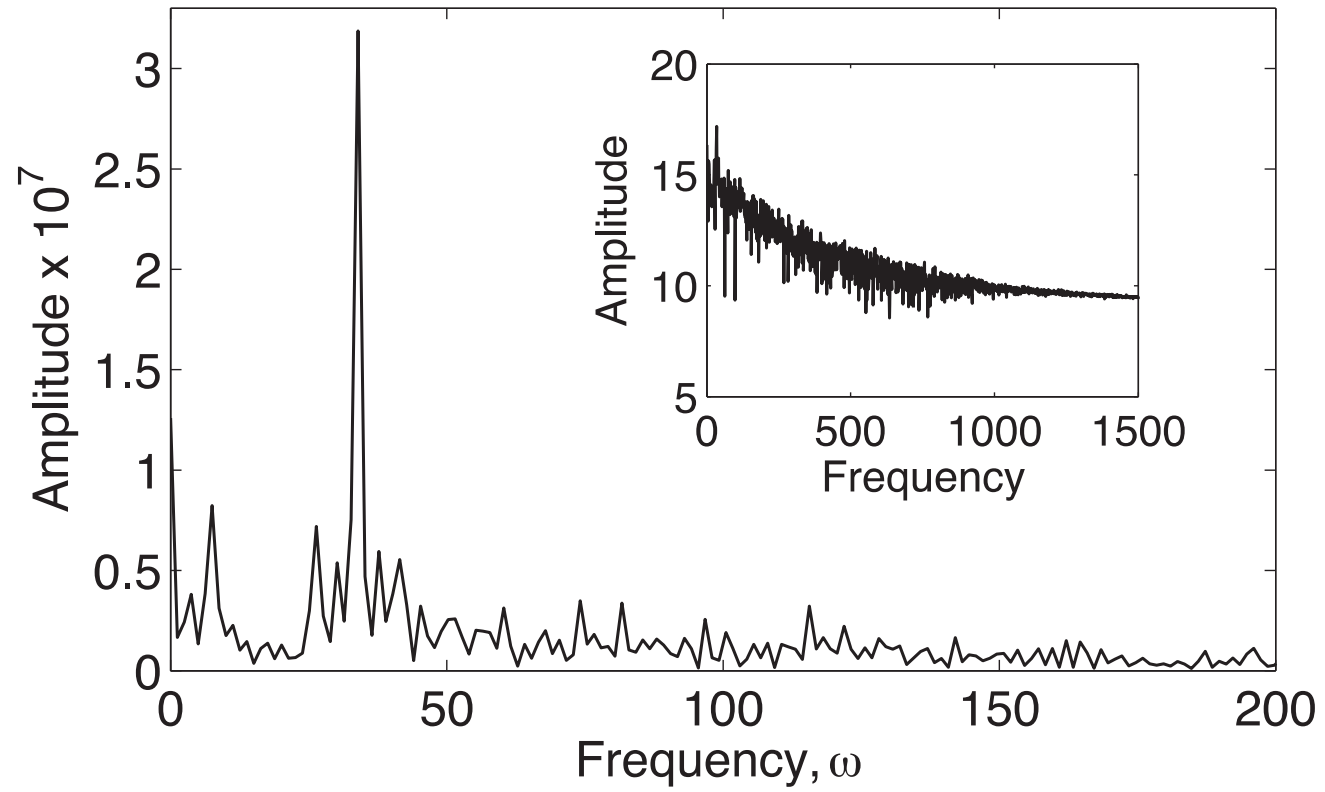


**Route to chaos → Ruelle-Takens-Newhouse scenario**

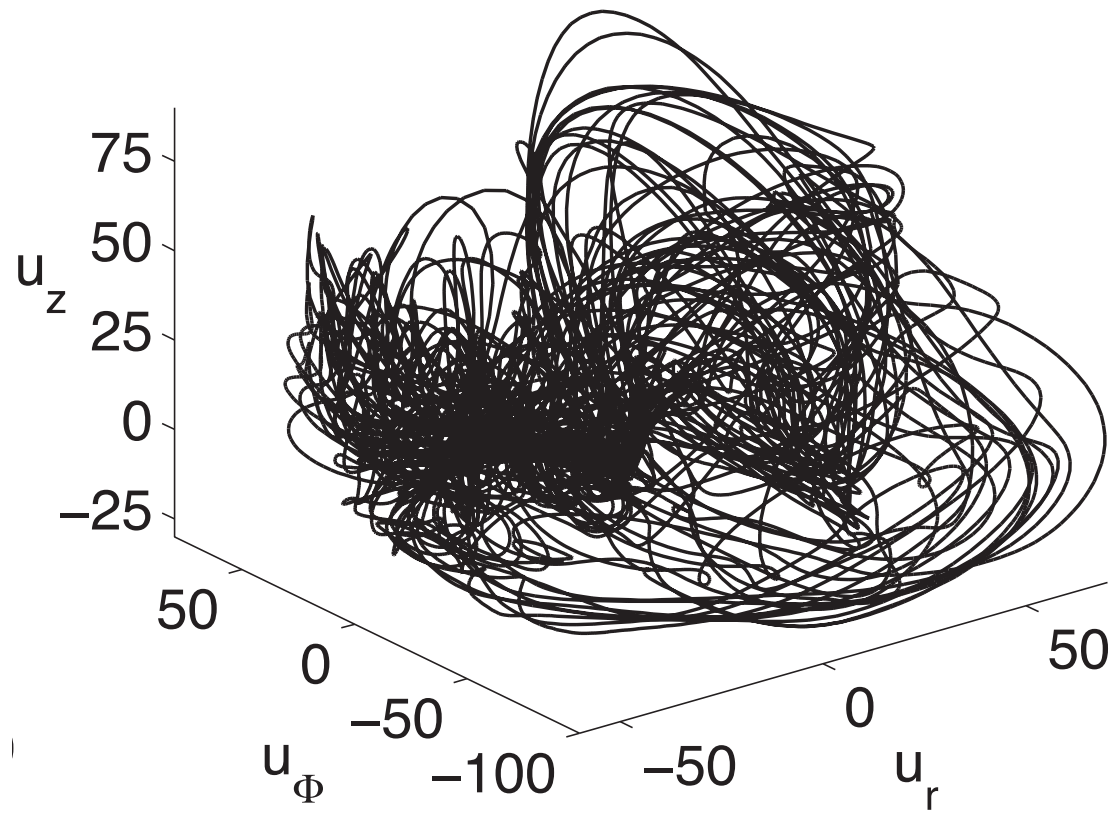
$Ra = 70000$



$Ra = 70000$



$Ra = 70000$



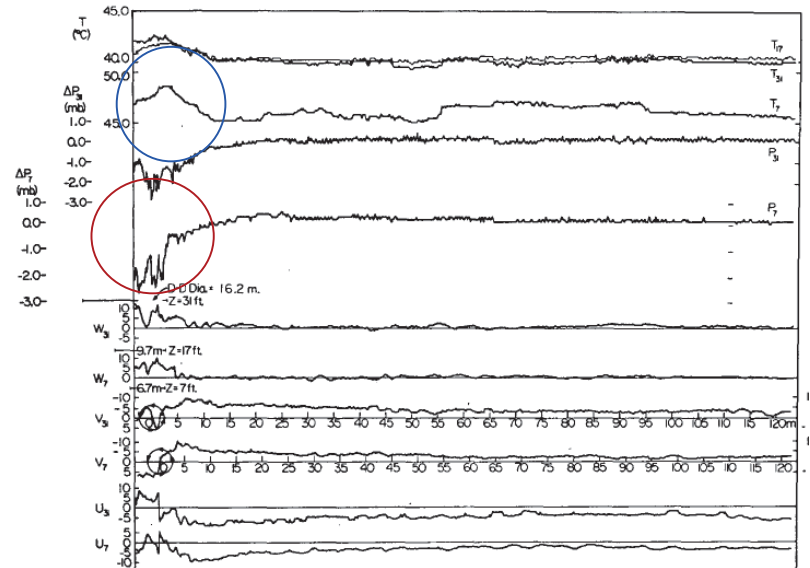
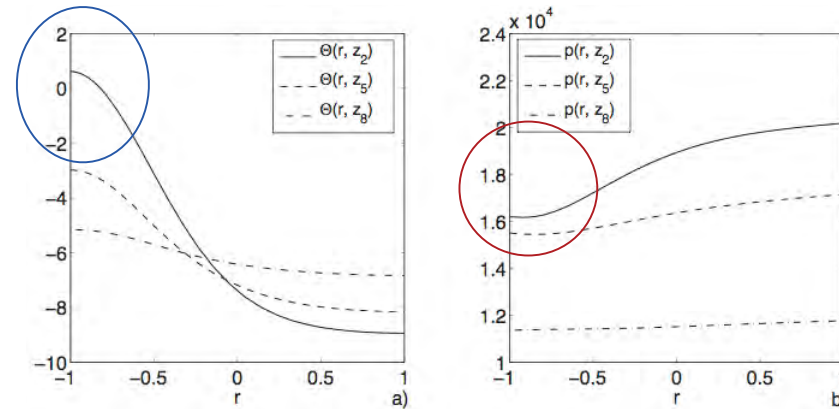
# Comparison observations dust devils

- Pressure, temperature and velocity fields
- Tilting of the axis of rotation
- Radius of maximum wind contraction
- Secondary whirls



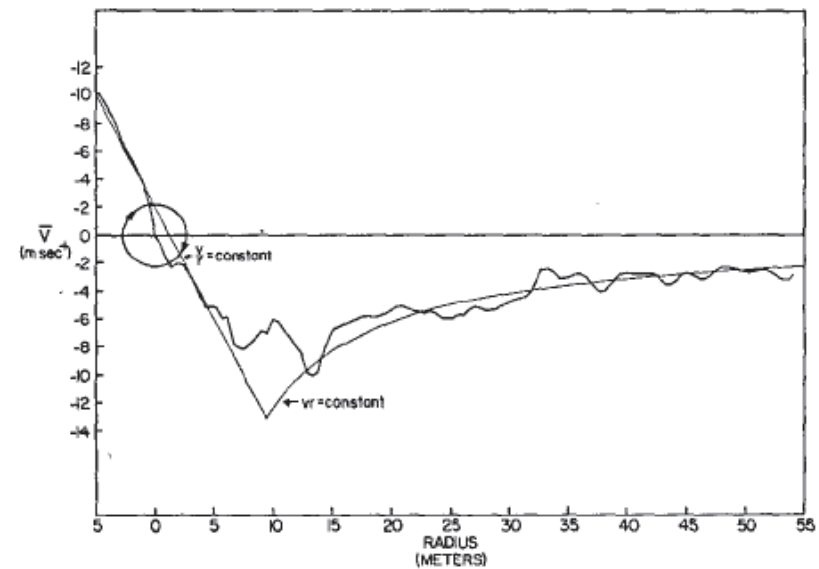
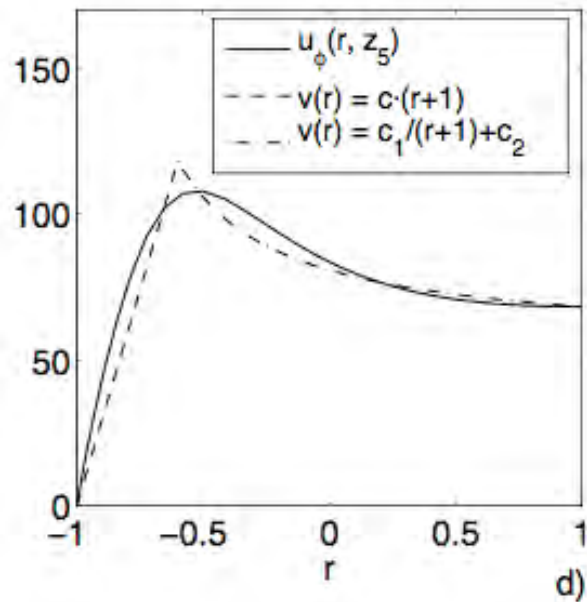
# Qualitative comparison with dust devils: pressure and temperature

- A low pressure region in the center
- Warm core

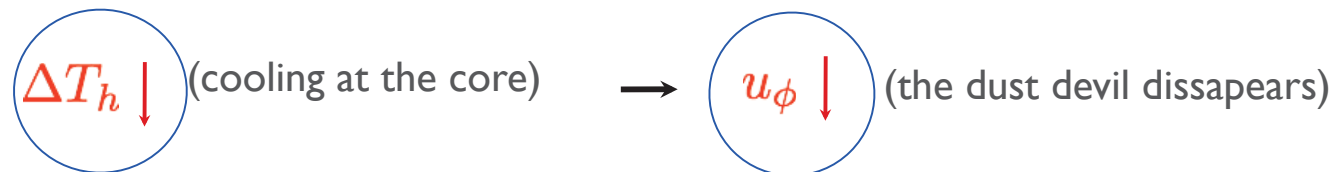
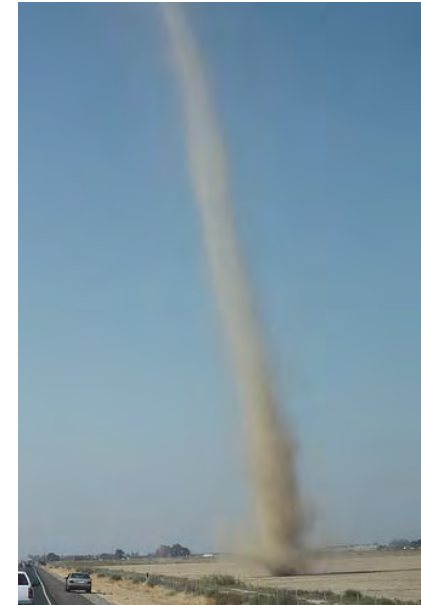
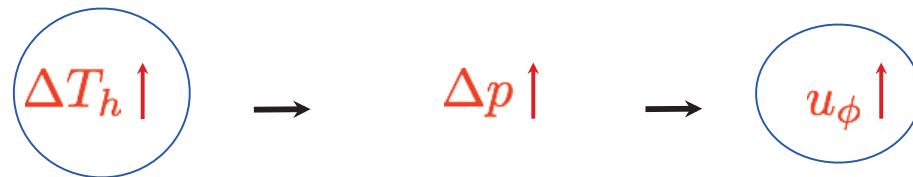


# Qualitative comparison with dust devils: tangential velocity

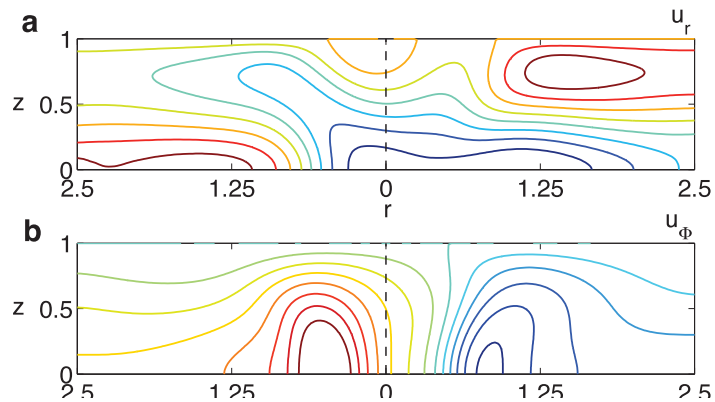
Rankine structure



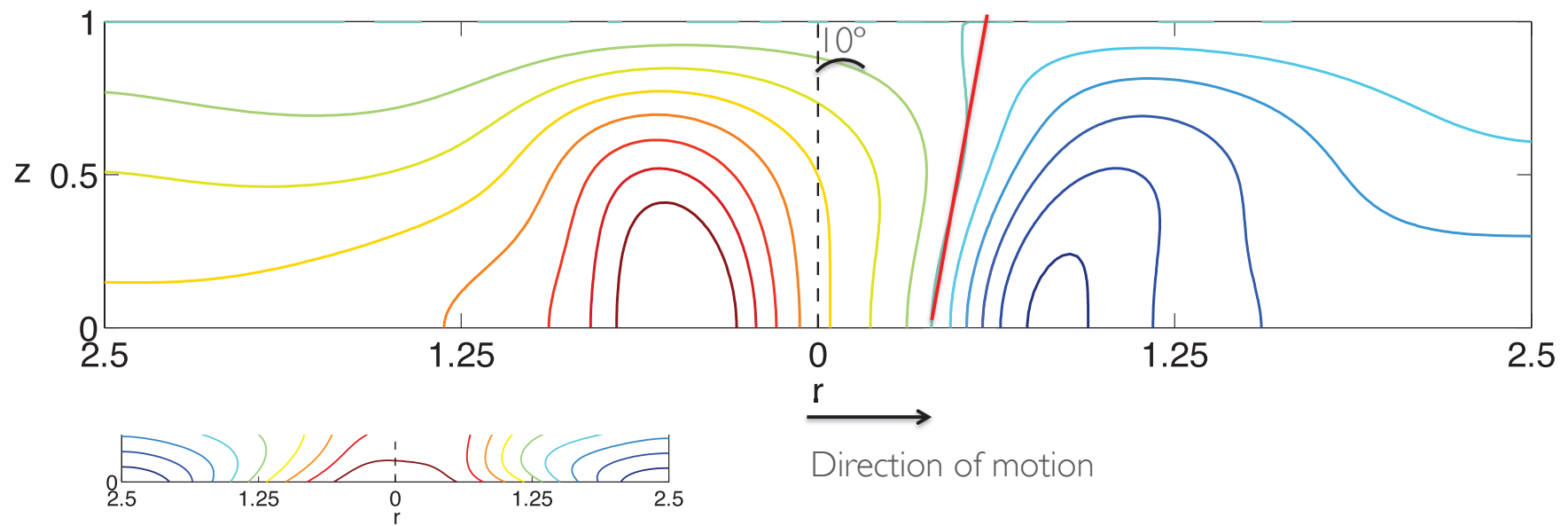
# Qualitative comparison with dust devils

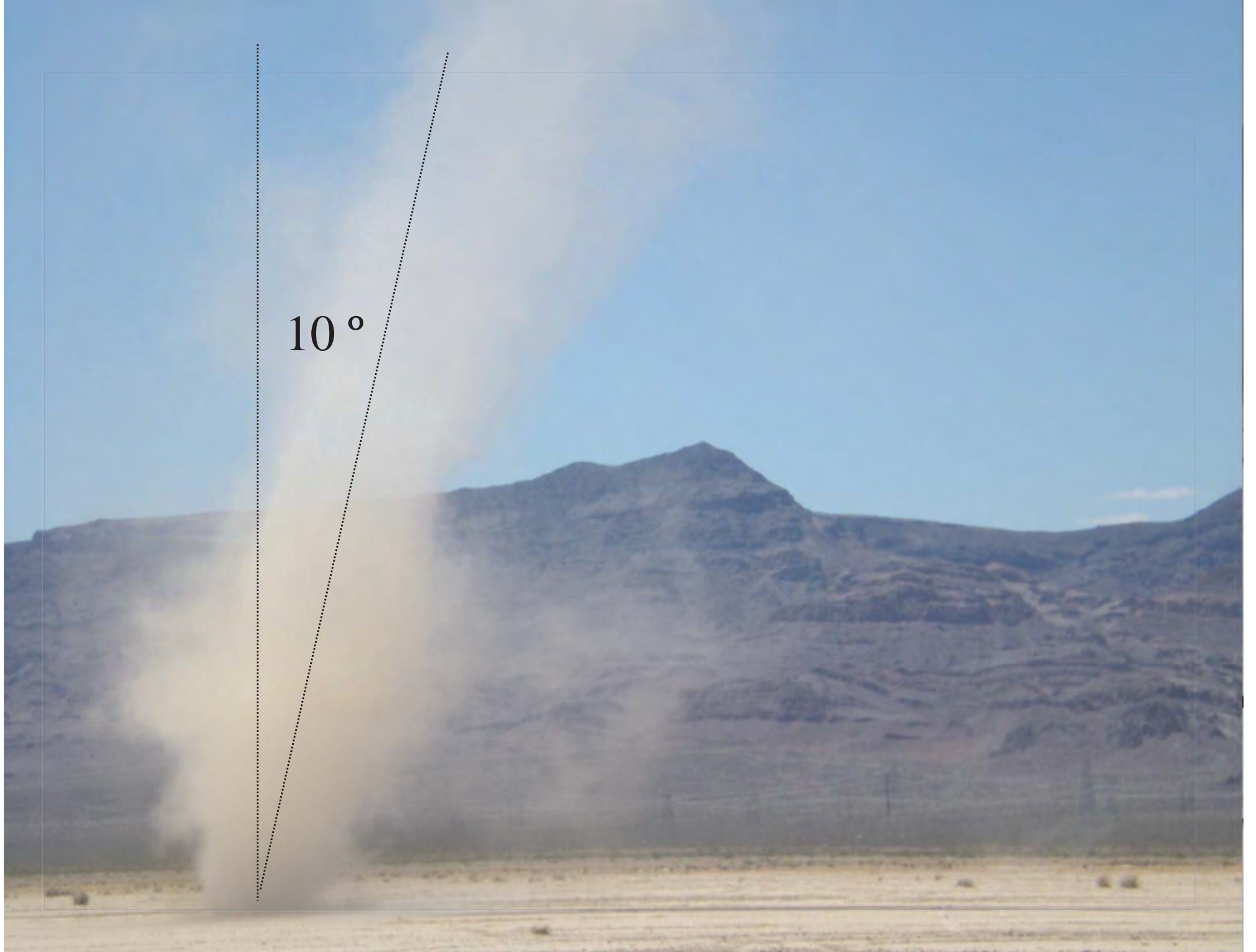


# Tilting

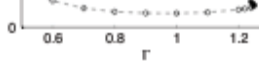


*Inclination  $10^\circ$*   
*-McGinnigle, Weather, 1966*  
*-Mattson, Weather, 1993*

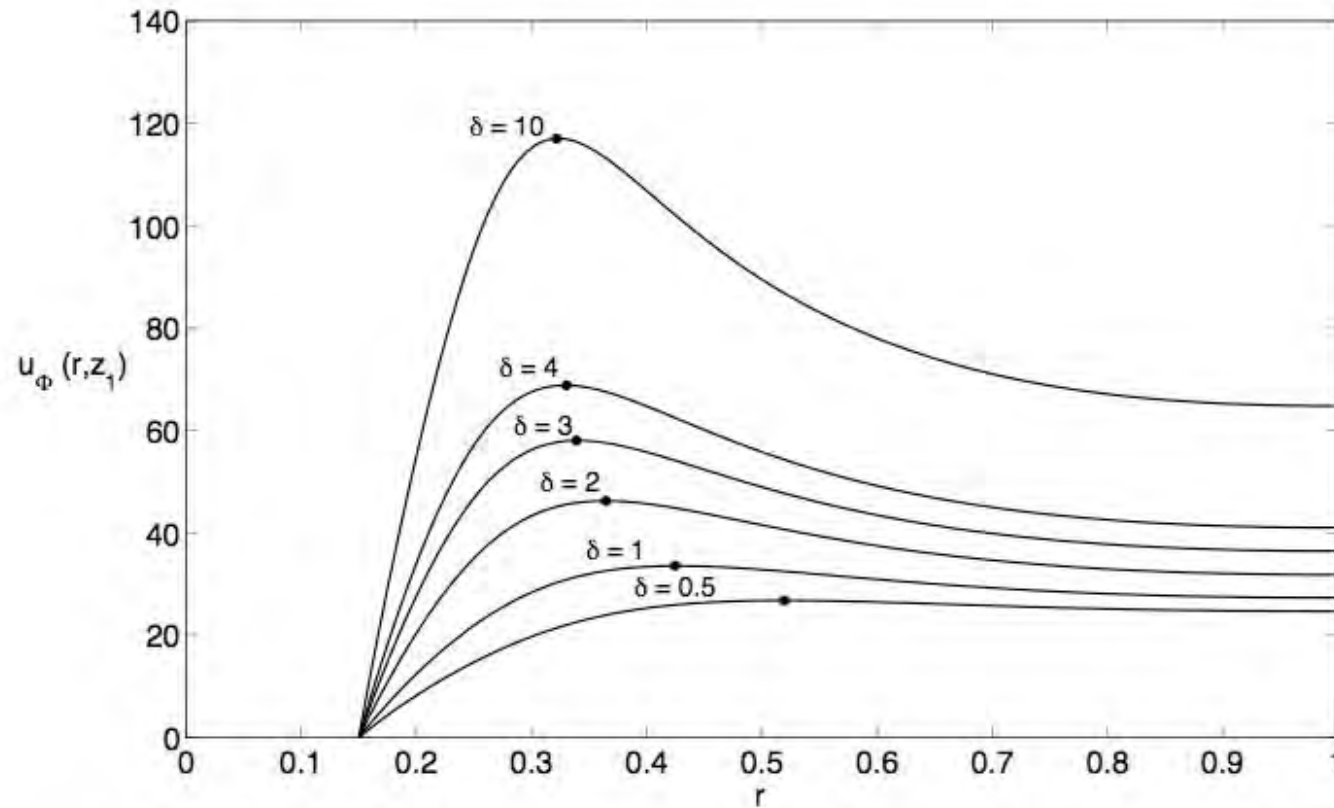




$10^\circ$



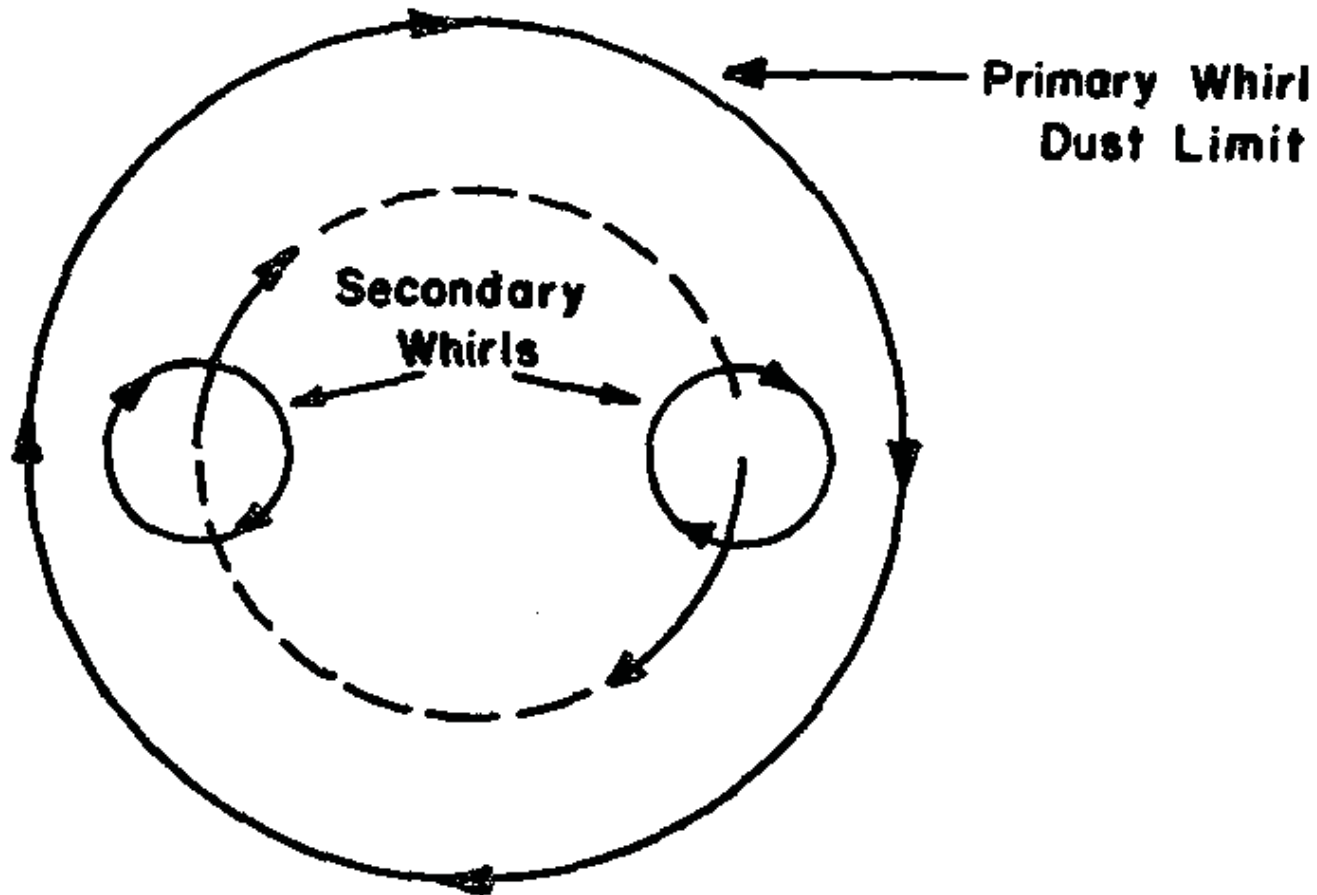
# RMW contraction

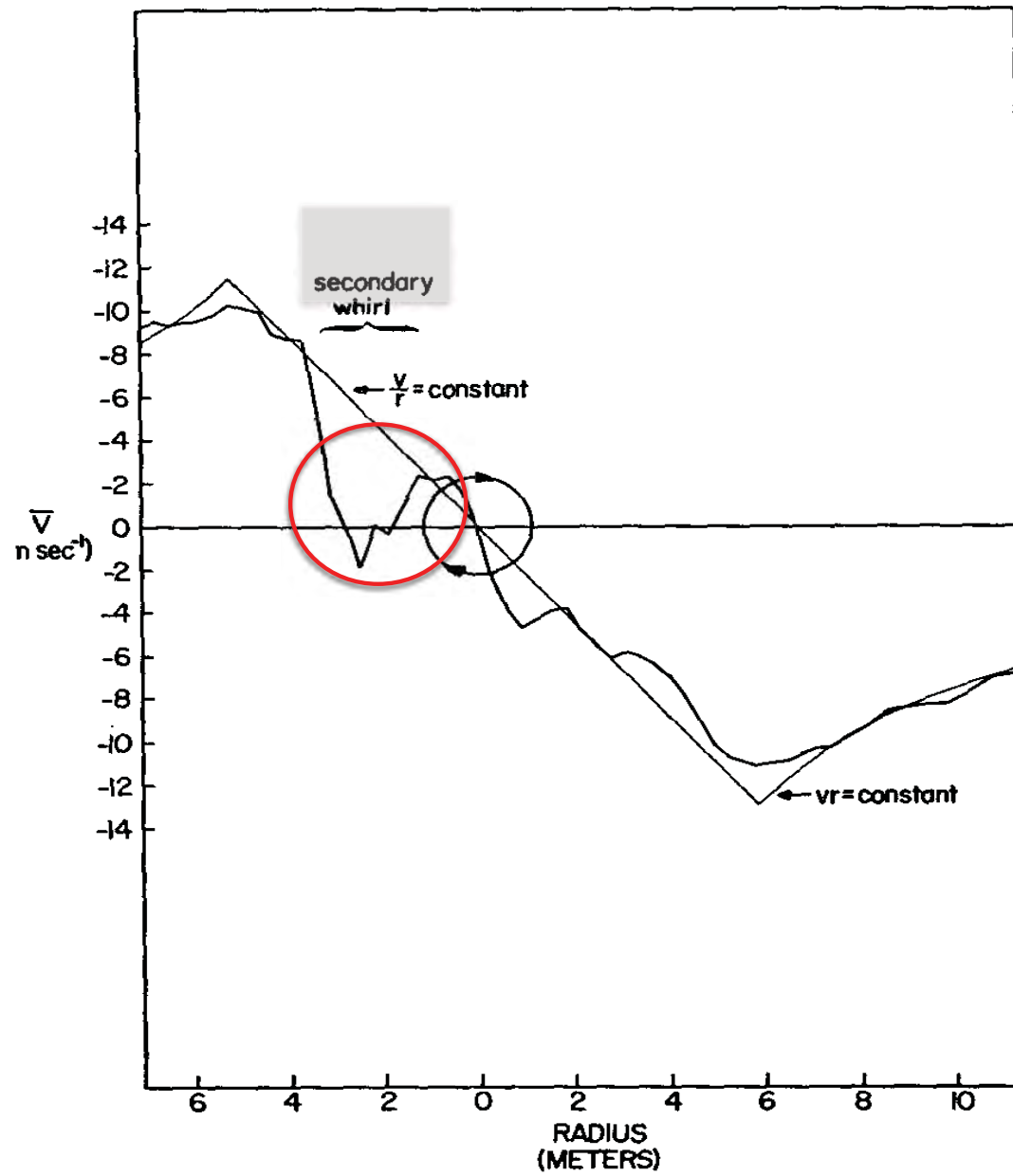


**Fig. 6.** (a) Critical Rayleigh number  $R_c$  and critical wavenumber  $k_c$  as a function of  $\delta$  for  $\Gamma = 0.5$ ; (b) critical Rayleigh number  $R_c$  and critical wavenumber  $k_c$  as a function of  $\Gamma$  for  $\delta = 1$ . Empty circles correspond to real eigenvalues while filled ones stand for complex eigenvalues.

# Secondary whirls

*Peter C. Sinclair, Journal of the Atmospheric Sciences 30, 1599-1619 (1973)*





(a)



# Conclusions

Vortical structures (spiral up motion) can be generated by a thermoconvective instability in a Rayleigh-Bénard problem (time dependent 3D numerical results)

- in a cylinder and in a cylindrical annulus
- heated non-homogeneously from below
- open outer boundary

Similar to dust devils and cyclones

- tilting
- RMW
- effect of thermal gradients
- secondary whirls

# Third International Workshop NONLINEAR PROCESSES IN OCEANIC AND ATMOSPHERIC FLOWS

Thank you for your attention

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