

# An eddy tracking algorithm from dynamical systems theory

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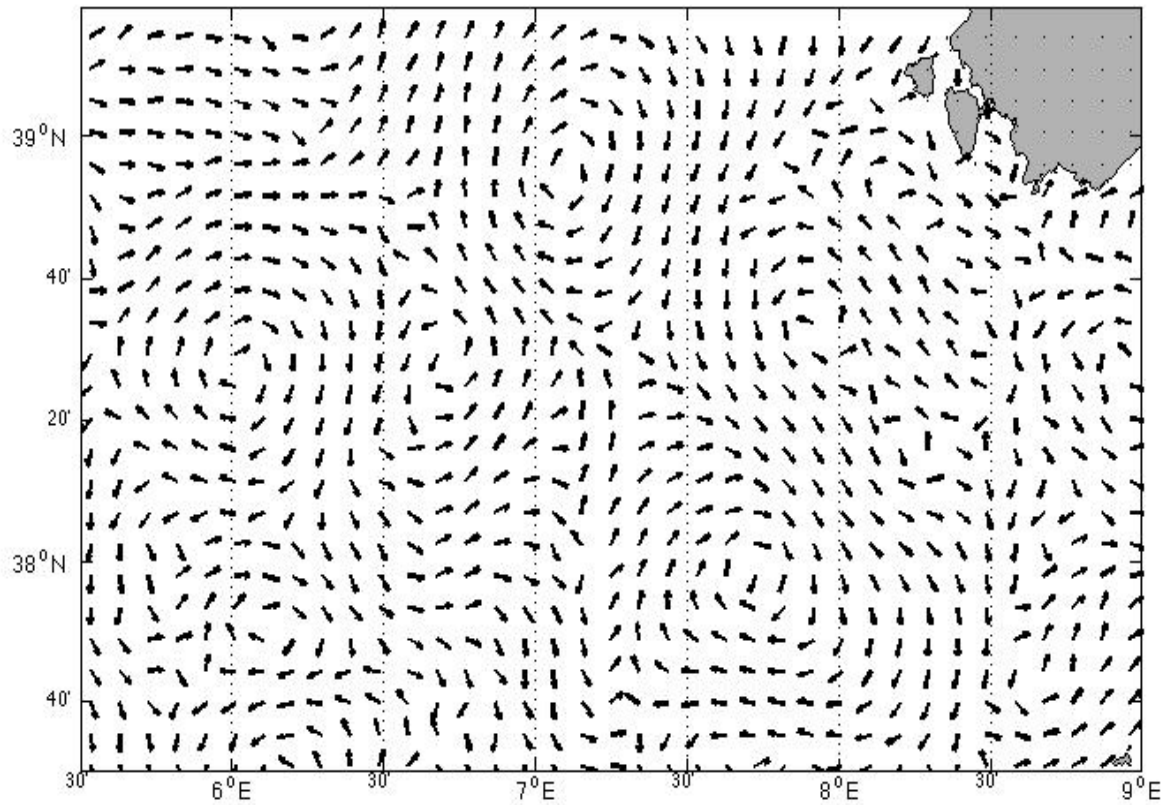


Third International Workshop

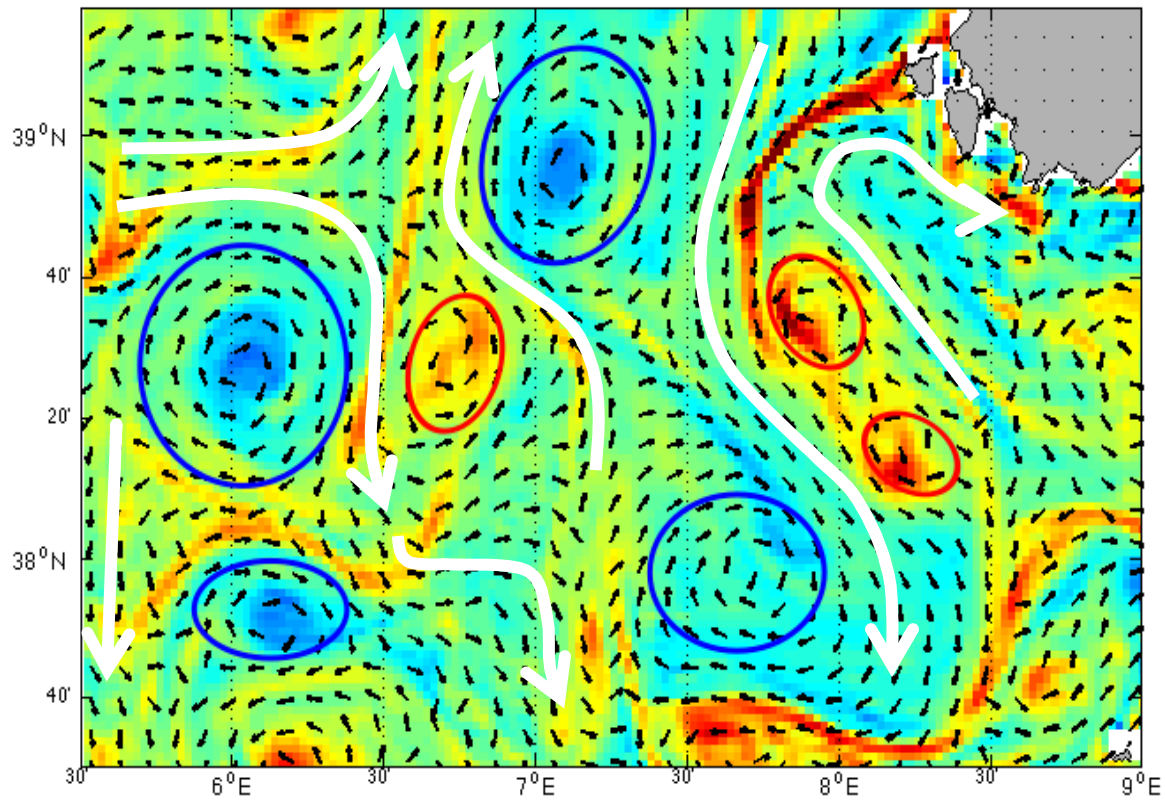
Universitat de les Illes Balears NONLINEAR PROCESSES IN OCEANIC AND ATMOSPHERIC FLOWS



# Motivation



# Motivation



# WHAT HAVE WE DONE?

A METHOD FOR IDENTIFYING AND  
TRACKING EDDIES FROM VELOCITY FIELDS

## HOW?

APPLYING THE THEORY OF 2D  
STATIONARY DYNAMICAL SYSTEMS

# Description of the Method

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1. Center of the eddies  
Stagnation Points and LSA

2. Size of the eddies  
Surrounding SP's

3. Link different times  
Tracking Algorithm

1. and 2.  
At a frozen time

# 1. Center of Eddies

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2D – steady flow

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}(\mathbf{r}, \tau_0) = \mathbf{v}_{\text{st}}(\mathbf{r})$$

Stagnation Points

$$\mathbf{v}_{\text{st}}(\mathbf{r}_0) = 0$$

Near the SP's

The velocity field is mainly dominated by the Jacobian Matrix

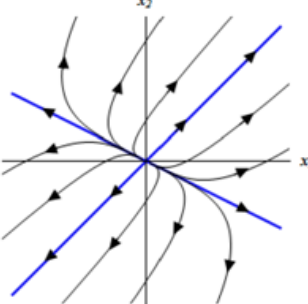
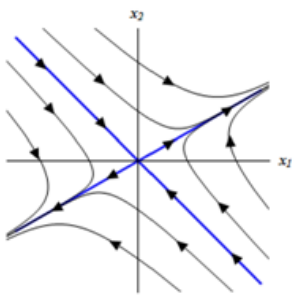
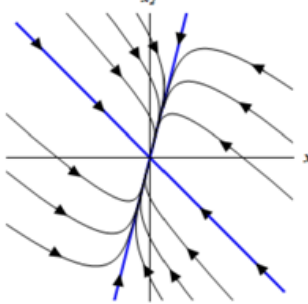
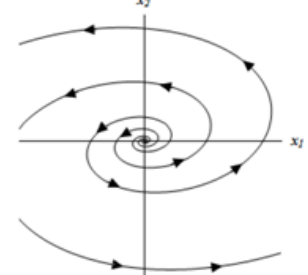
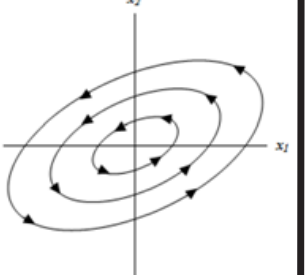
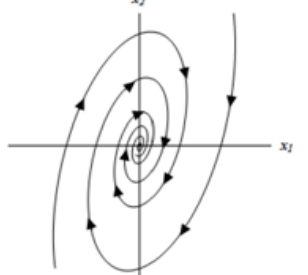
$$\mathbf{v}_{\text{st}}(\mathbf{r}_0 + \delta\mathbf{r}) = \mathcal{J}(\mathbf{r}_0) \cdot \delta\mathbf{r}$$

$$\mathcal{J}(\mathbf{r}_0) = \left( \begin{array}{cc} u_x & u_y \\ v_x & v_y \end{array} \right) \Big|_{\mathbf{r}_0}$$

# Linear Stability Analysis

$$\lambda_{1,2} = \frac{1}{2} \left( T_J \pm \sqrt{T_J^2 - 4\Delta_J} \right)$$

The eigenvectors-eigenvalues of the Jacobian matrix allow us to know the behaviour around the stagnation points

	$\text{Re}(\lambda_{1,2}) > 0$	$\text{Re}(\lambda_1) > 0 > \text{Re}(\lambda_2)$	$\text{Re}(\lambda_{1,2}) = 0$	$\text{Re}(\lambda_{1,2}) < 0$
$\lambda_{1,2} \in \mathbb{R}$	unstable node 	saddle node 		stable node 
$\lambda_{1,2} \in \mathbb{C}$	unstable spiral 		center 	stable spiral 

# Linear Stability Analysis

$$\lambda_{1,2} = \frac{1}{2} \left( T_{\mathcal{J}} \pm \sqrt{W} \right)$$

$$W = \underline{\sigma^2} - \underline{\omega^2}$$

$W$  Okubo – Weiss parameter  
deformation – vorticity

$W < 0 \rightarrow$  Eddy Center

$$T_{\mathcal{J}} = u_x + v_y \equiv \nabla \cdot \mathbf{v}_{st}$$

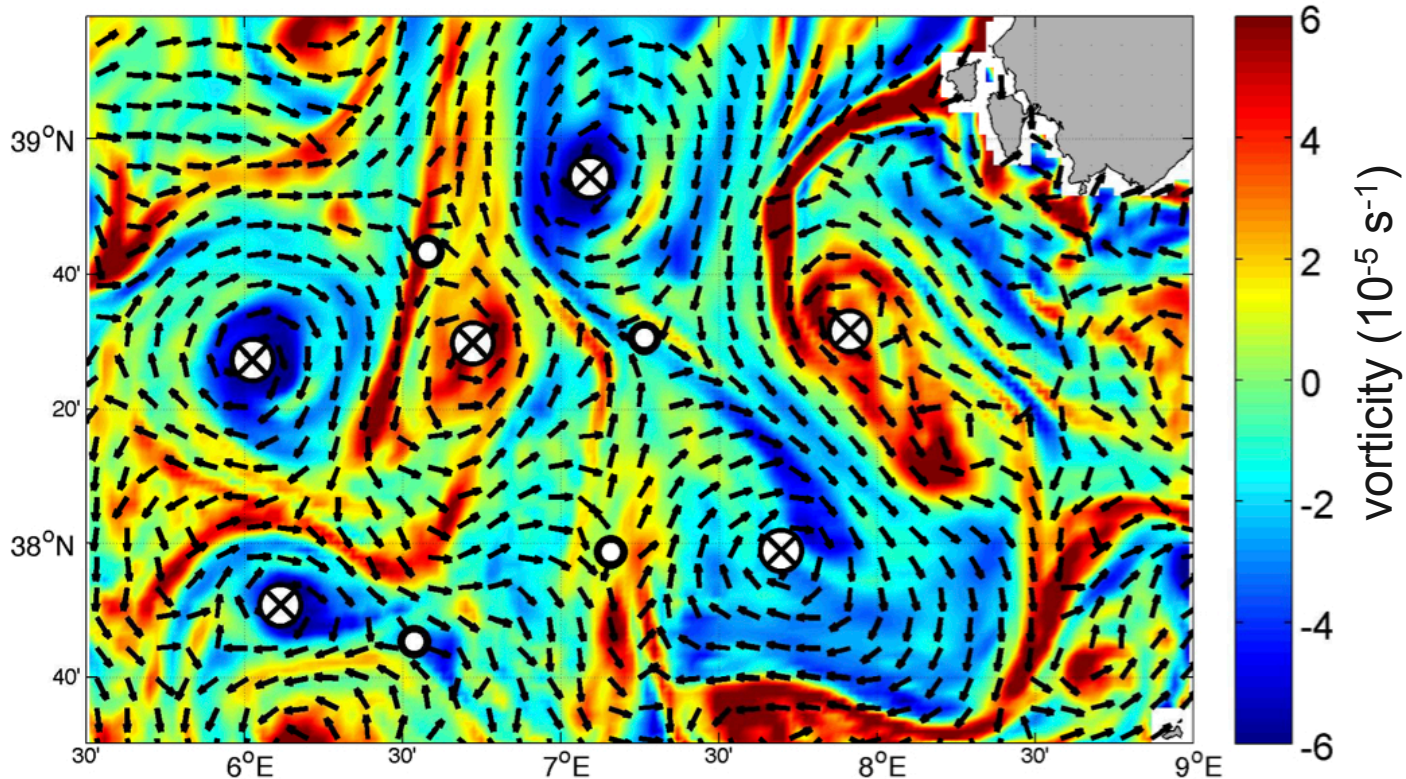
For incompressible 2D flux  
 $\rightarrow$  Only Saddle Nodes & Centers



# Example

Eddy Center ⊗

Saddle Node ○



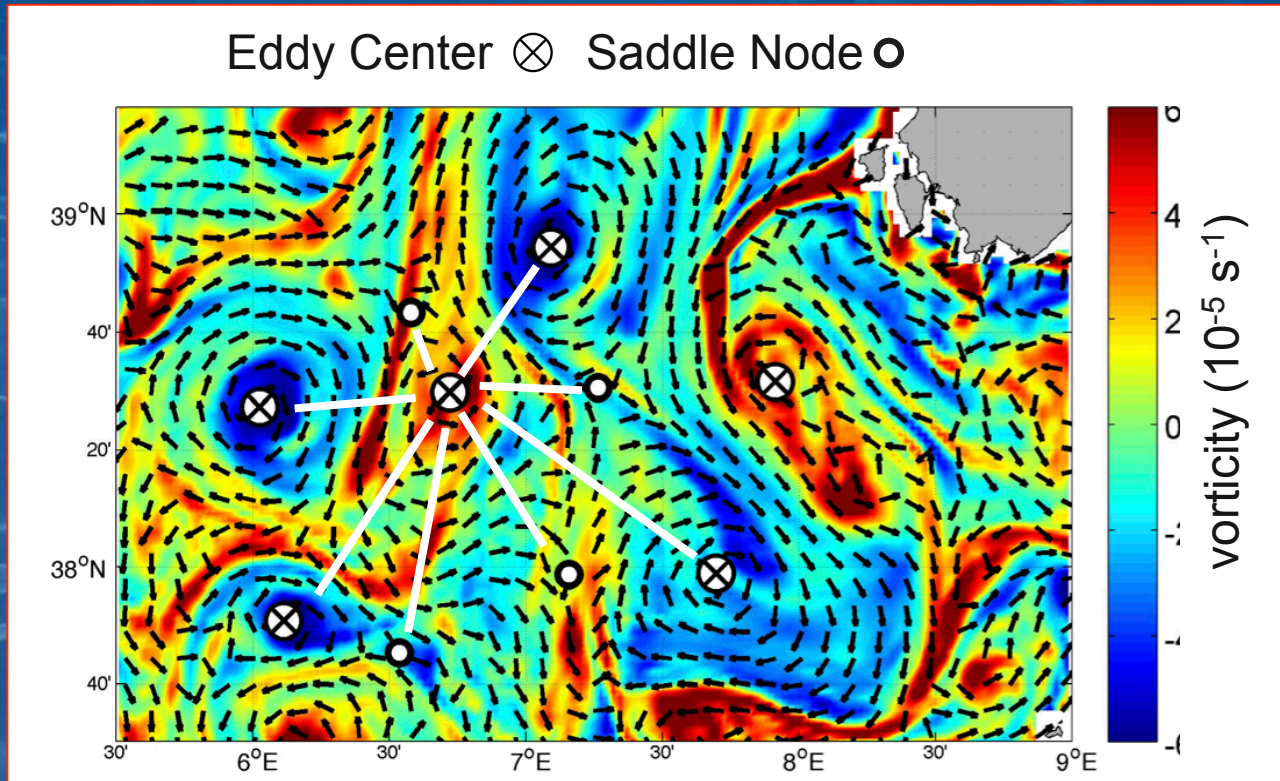
ROMS (WMOP regional configuration)

Model Resolution  $\sim 2 \text{ km}$

Area: South-West of Sardinia

Date: 2015-05-02

## 2. Size of the Eddies

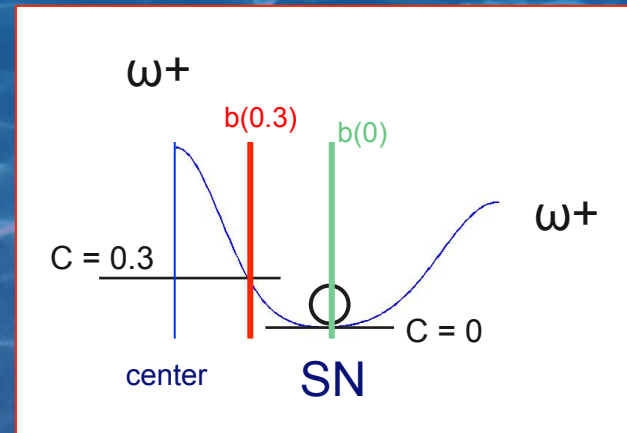
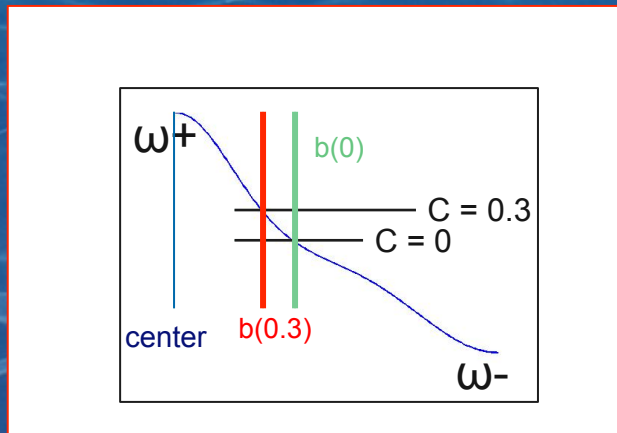


Stationary Field with eddy vortices + saddle nodes

Eddy Boundaries are estimated evaluating the vorticity along the white lines

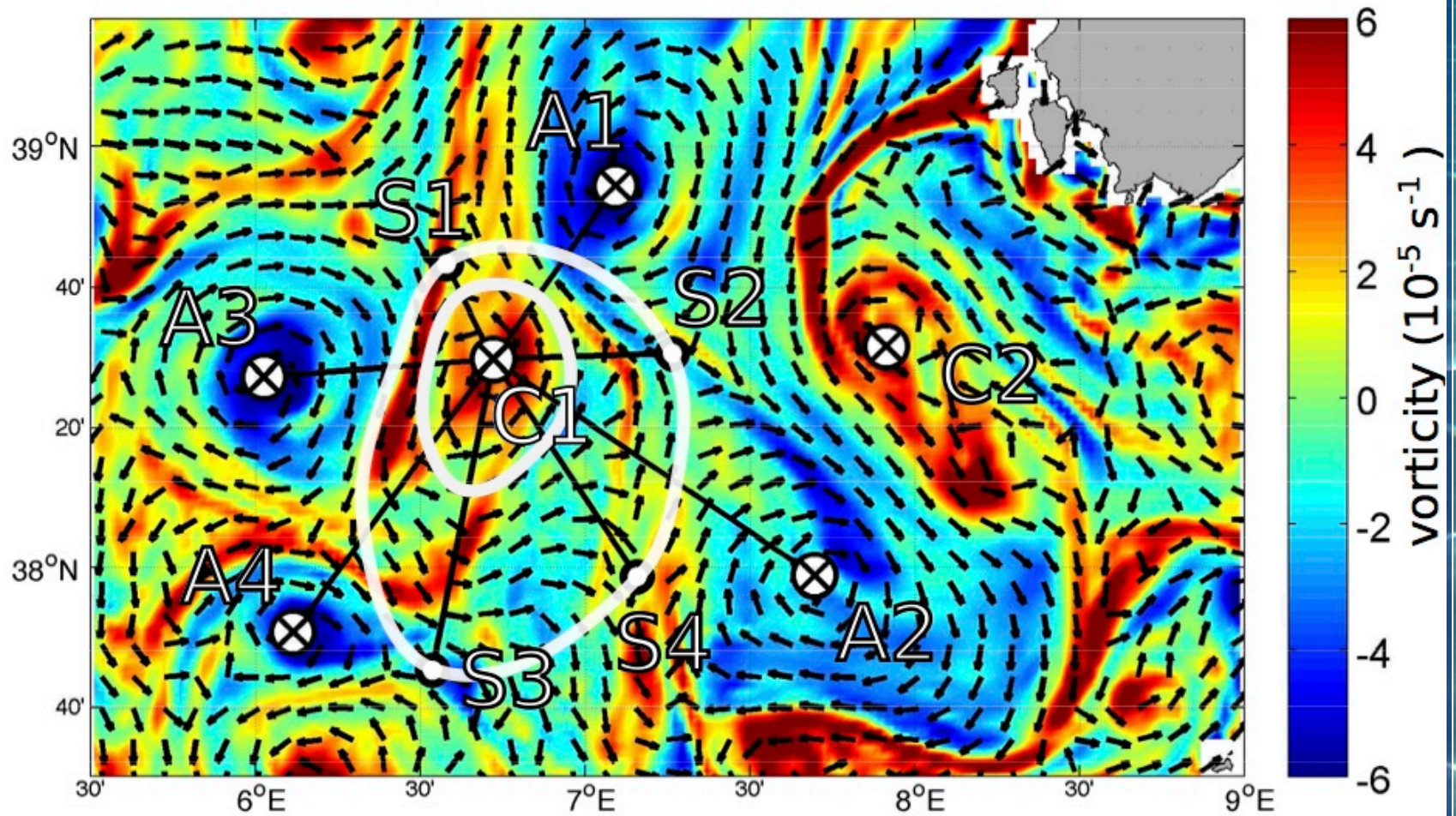
## 2. Size of the Eddies

Vorticity profiles along the connecting lines



We define eddy boundary as the point in the connecting line where  $\omega = c \omega_+$

## 2. Size of the Eddies



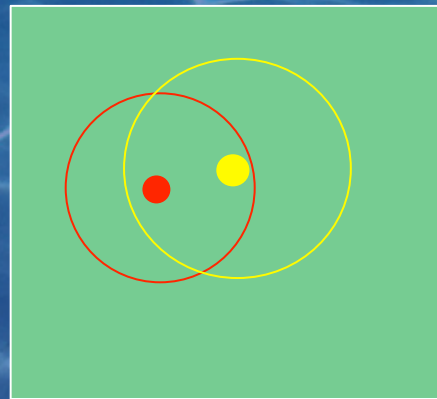
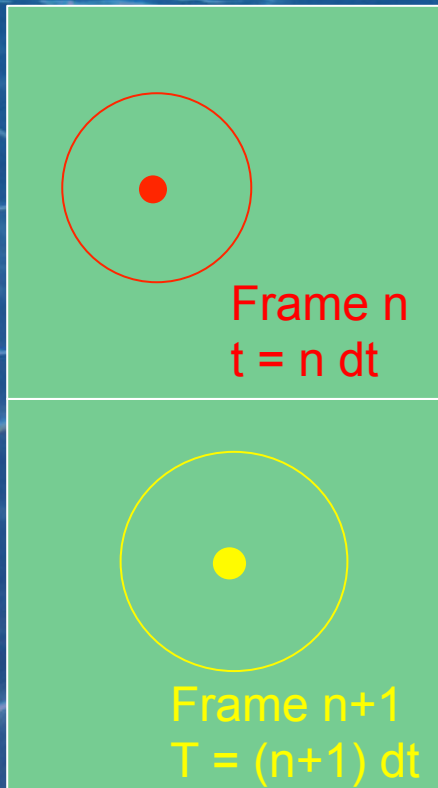
# 3. Tracking algorithm

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We have eddy data at a discrete sequence of times

Relate the eddy data in each snapshot with his previos and next snapshots

Decide the correspondence between eddies of consecutive frames

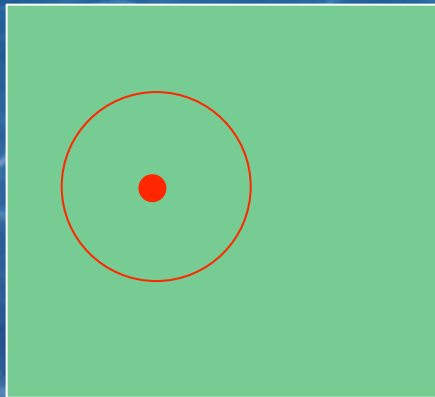
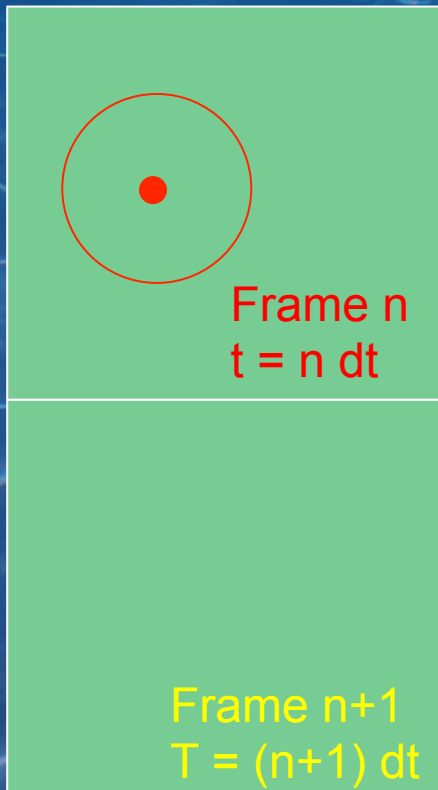


It is the same eddy  
The eddy has moved  
His radio has increased

# 3. Tracking algorithm

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Relate the eddy data in each snapshot with his previous and next snapshots  
Decide the correspondence between eddies at consecutive frames

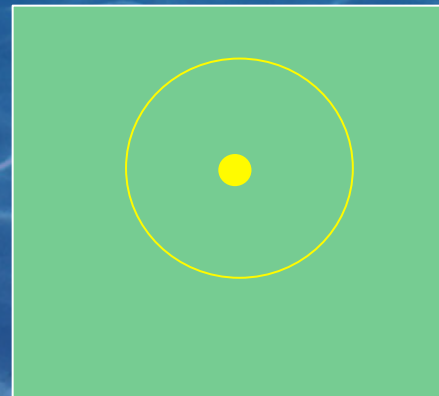
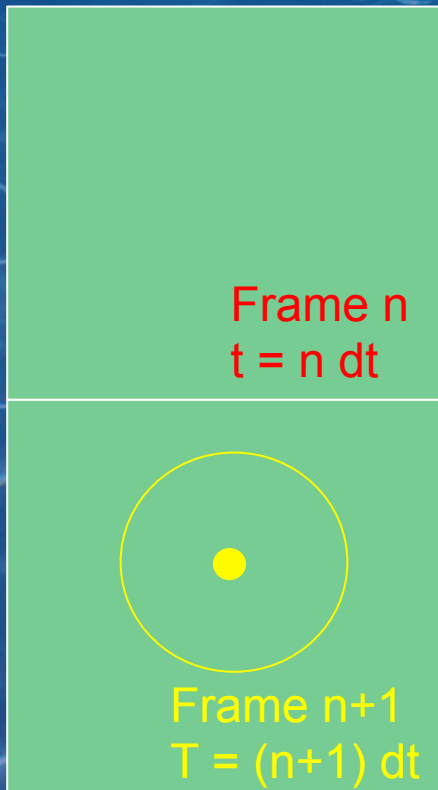


The eddy has finished his life

# 3. Tracking algorithm

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Relate the eddy data in each snapshot with his previos and next snapshots  
Decide the correspondence between eddies at consecutive frames

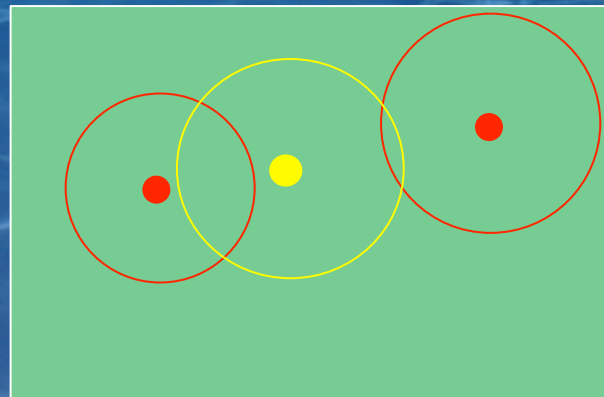
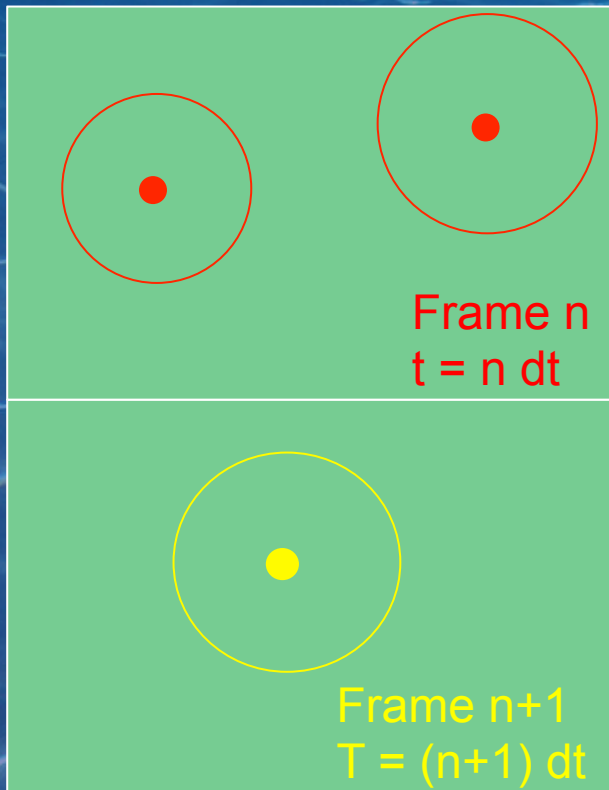


An eddy has born

# 3. Tracking algorithm

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Relate the eddy data in each snapshot with his previos and next snapshots  
Decide the correspondence between eddies at consecutive frames



???

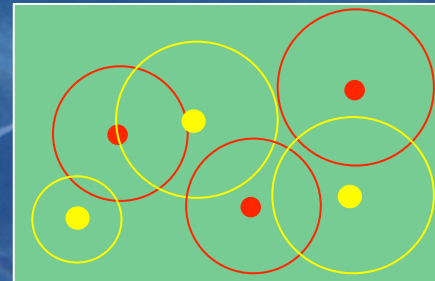
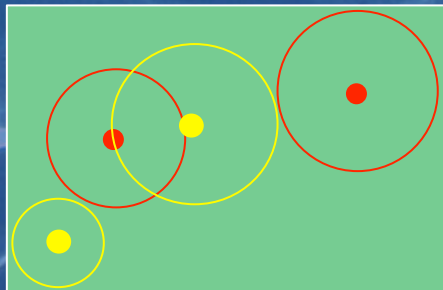


# 3. Tracking algorithm

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Two main problems:

- a) Loosing the signature of and eddy in a single frame  
Sampling errors and/or noisy data from measurements or  
inhability of the method to detect eddies  
Leads to artificial dead + born of eddies
  
- b) Mismatching connections  
Wrong decision in the eddy matching between frames  
Coupled configurations



# 3. Tracking algorithm

---

Two main problems:

- a) Loosing the signature of and eddy in a single frame
  - Sampling errors and/or noisy data from measurements
  - Inhability of the method to detect eddies
  - Leads to artificial dead + born of eddies

- b) Missmatching connections
  - Wrong decision in the eddy matching between frames
  - Coupled configurations



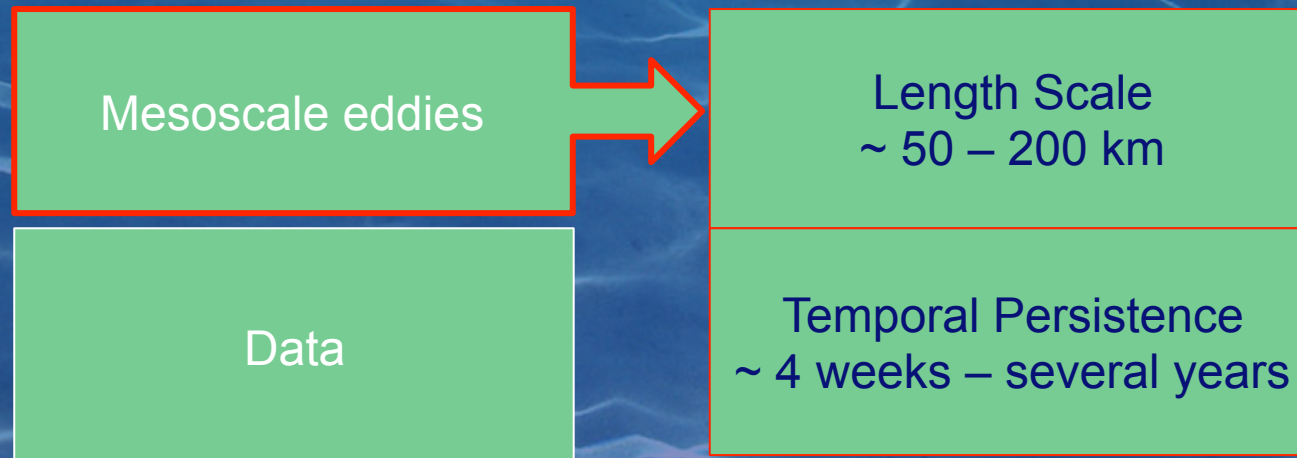
Collective minimization of distances of all the possible pairments

# Search for Mesoscale Eddies

Mesoscale eddies

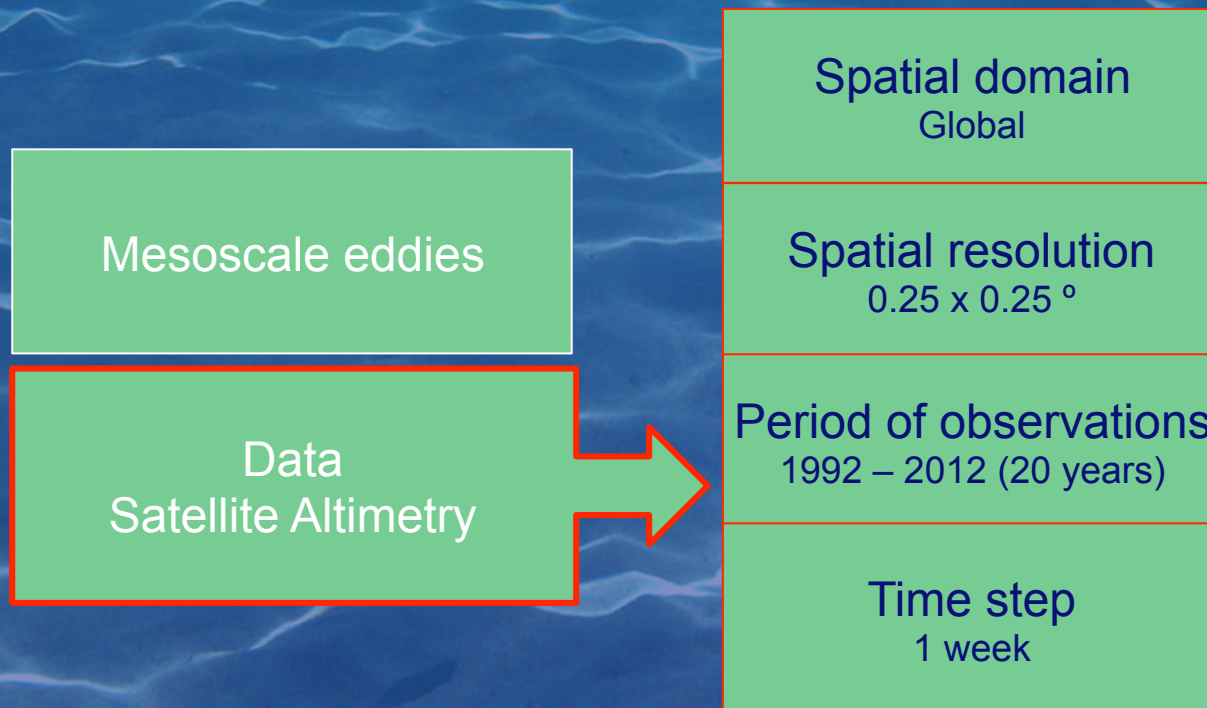
Rotatory CS's present in all world oceans.

Important role in regulating the global climate and transport processes



# Search of Mesoscale Eddies

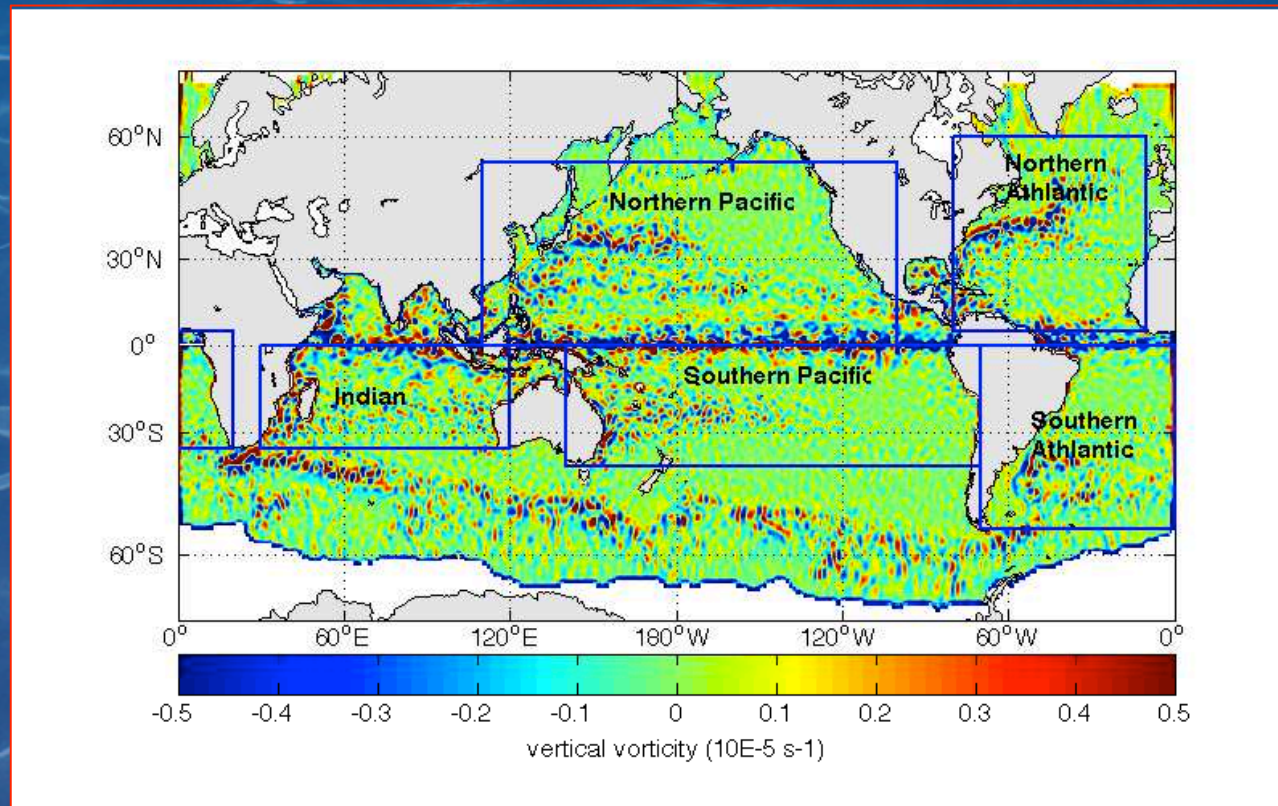
Data Used: Geostrophic velocities derived from Satellite Altimetry Dataset of 20 years of weekly SSH data from AVISO product <http://www.aviso.altimetry.fr/>



# Geostrophic Balance

It is the *steady state* of a flow in the Earth Rotating System, where pressure gradients are balanced with the Coriolis force

$$u(x, y) = -\frac{g}{f} \frac{\partial H}{\partial y}$$
$$v(x, y) = \frac{g}{f} \frac{\partial H}{\partial x}$$



Snapshot of vorticity field of Geostrophic Velocities from AVISO SSH

# Results(i)

[www.researchgate.net](http://www.researchgate.net)



**Dataset** · January 2016

DOI: 10.13140/RG.2.1.1492.5682



1st [Daniel Conti Sampol](#)

|| 12.88 · Mediterranean Institute for Advanced S...



2nd [Alejandro Orfila](#)

|| 34.26 · Spanish National Research Council

## Abstract

Dataset associated to the article 'An eddy tracking algorithm from dynamical systems theory', Ocean Dynamics, which is under revision.

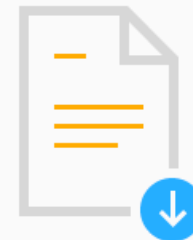
eddies\_19921014\_20120208.nc

21.22 MB

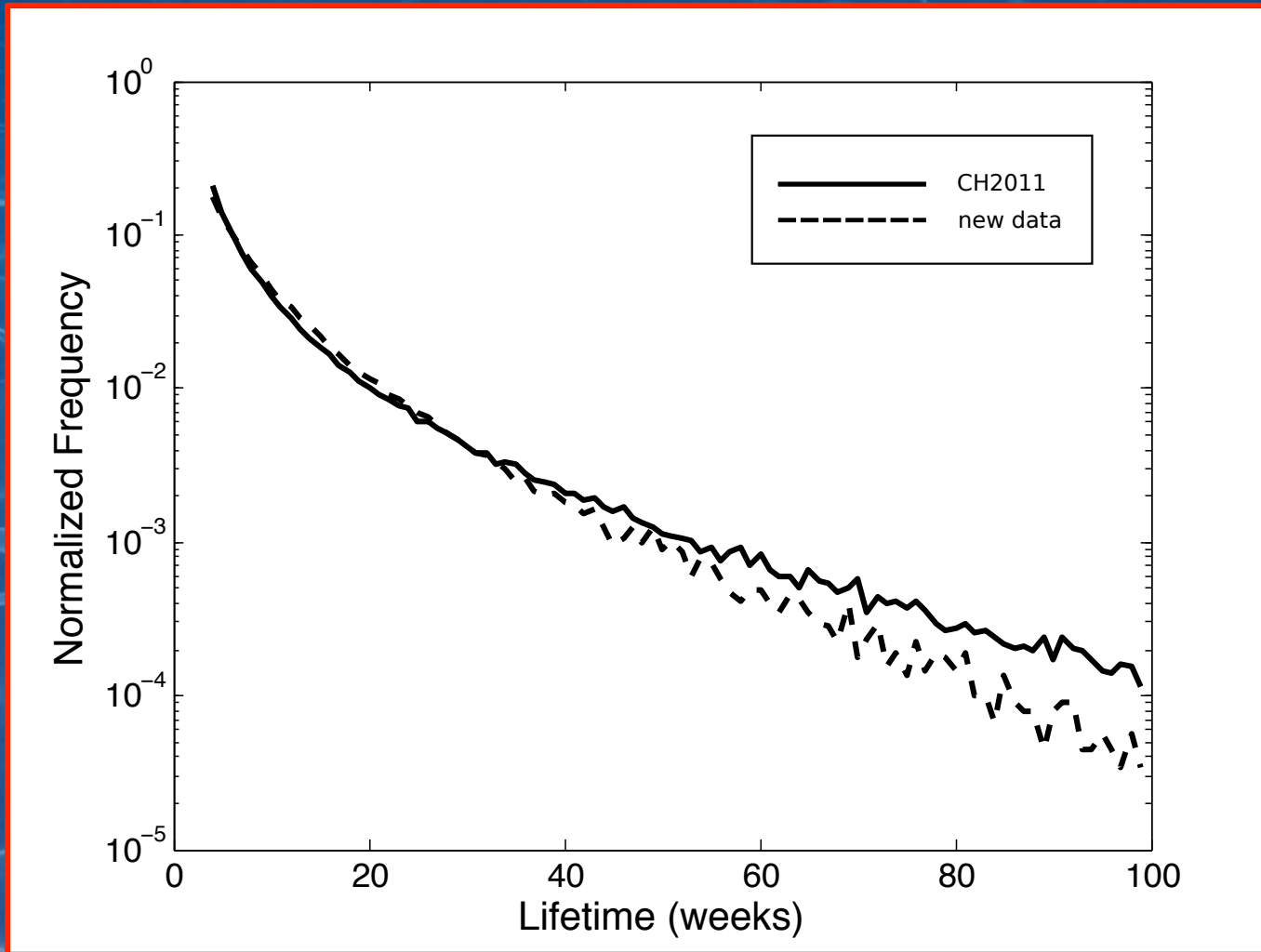
Sorry, there is no online preview for this file type.

To view this dataset, please download it below.

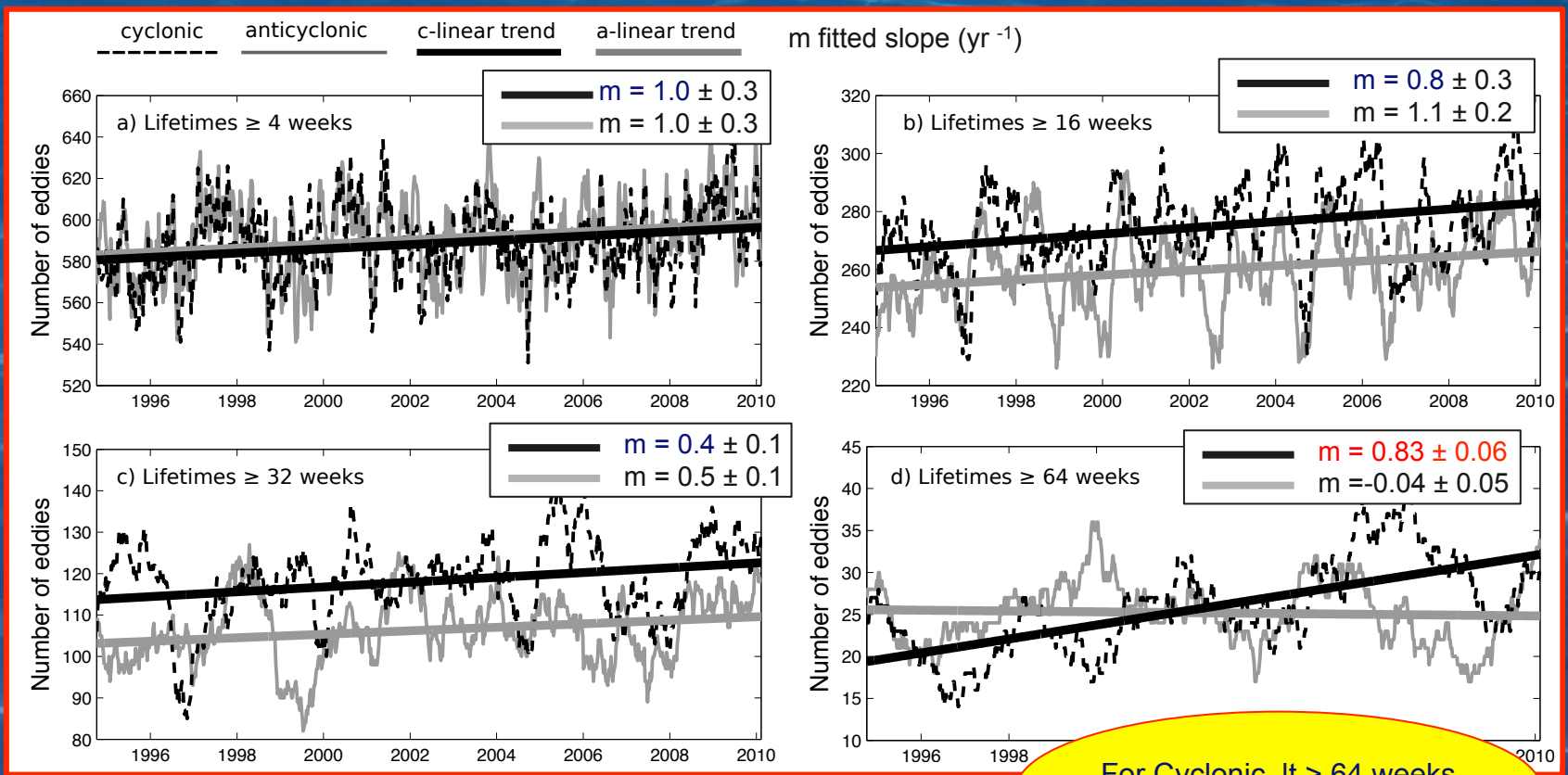
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# Results (ii) Lifetimes



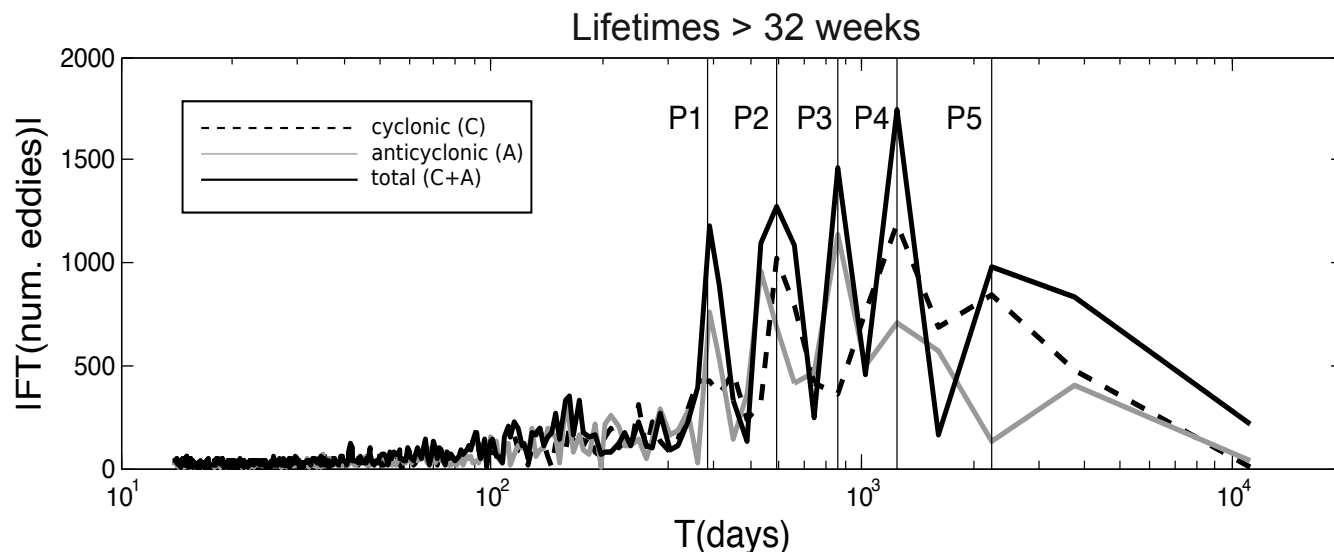
# R (ii) TimeSeries



For Cyclonic, It > 64 weeks  
 $m/\langle n \rangle = 3\%$  yearly!



# R (iii) Spectra



[P1, P2, P3, P4, P5] = [1.0, 1.6, 2.4, 3.4, 6.1] years

# Summary

## Method

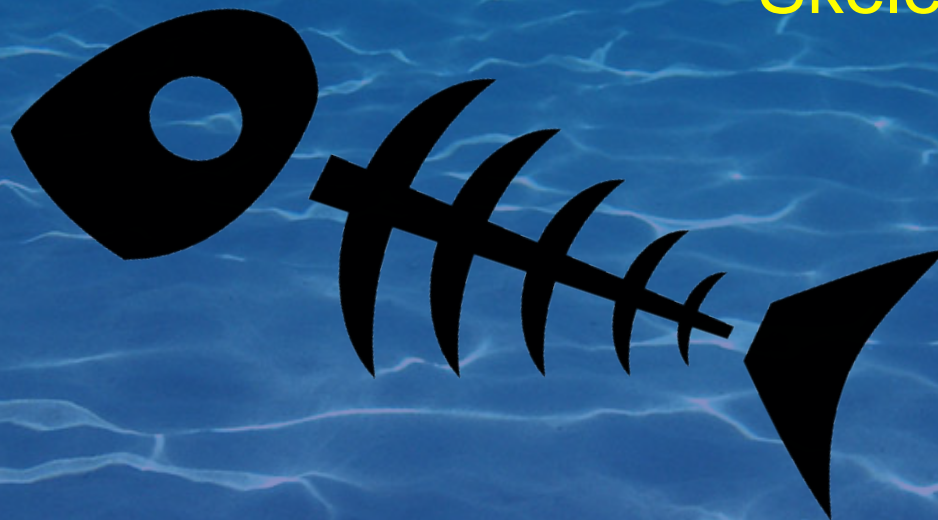
- Eddies are detected as Eulerian structures at frozen times and then tracked
- The eddy center and size are estimated from stagnation points
- The tracking method improves the existing methods at the decision making of pairments when coupled configurations are involved

## Mesoscale Eddies

- The temporal and spatial scales of mesoscale eddies allows to represent them as time-evolving Eulerian structures
- We provide database of 20 years of eddies in the global ocean
- The database compares qualitatively well against other global databases in terms of lifetimes, latitudinal distribution, density distribution



## “Discovering the Skeleton of the flow”



Thank you for your attention