Variational Destriping on Remote Sensing Imagery

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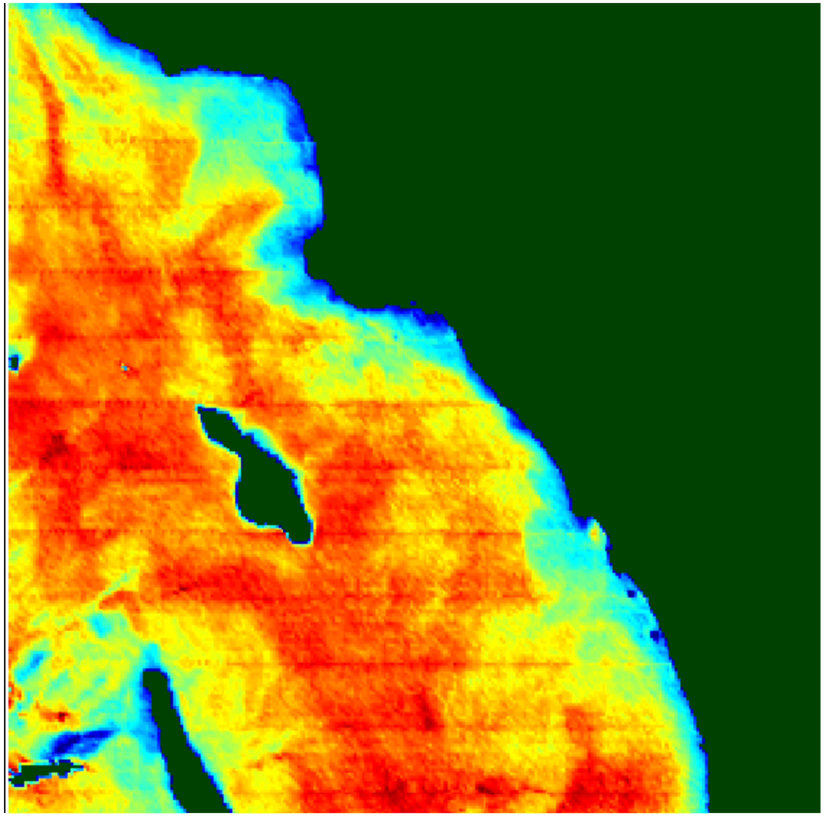
Suomi National Polar-orbiting Partnership (Suomi NPP)

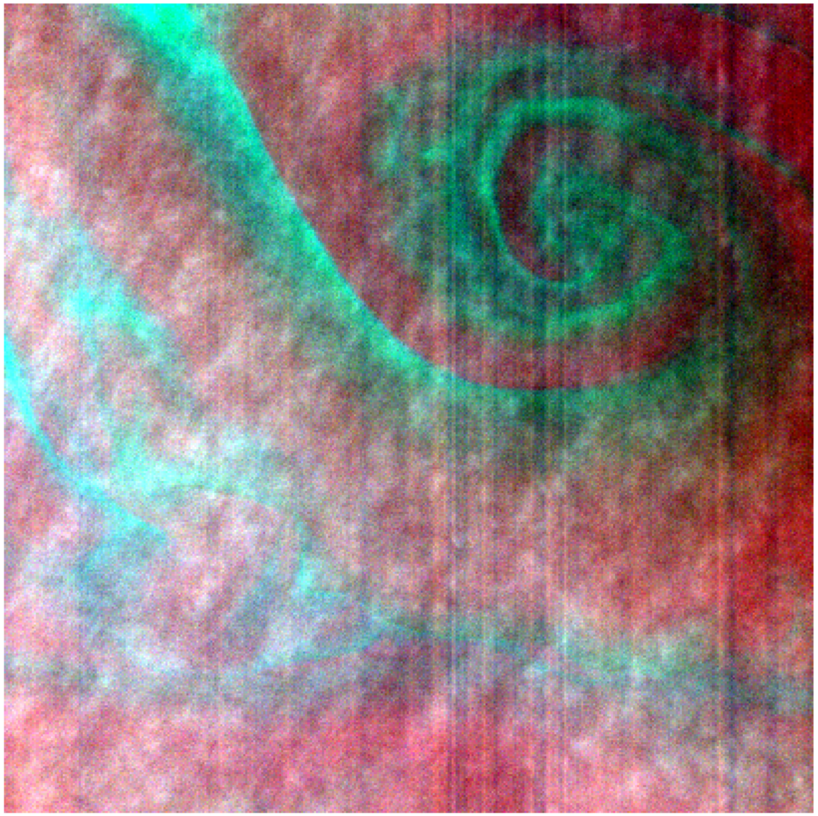
Visible Infrared Imaging Radiometer Suite (VIIRS)

- A scanning radiometer
- Features daily multi-band imaging
- Collects visible and infrared imagery
- Generate 22 spectral bands
- Collects global observations of Earth's clouds, atmosphere, oceans and land surfaces
- Produces higher-resolution and more accurate measurements of sea surface temperature
- Has 16 detectors

The Problem: Stripes



VIIRS Image



HICO (Hyperspectral Imager for the Coastal Ocean) Image

Affects and the Solution

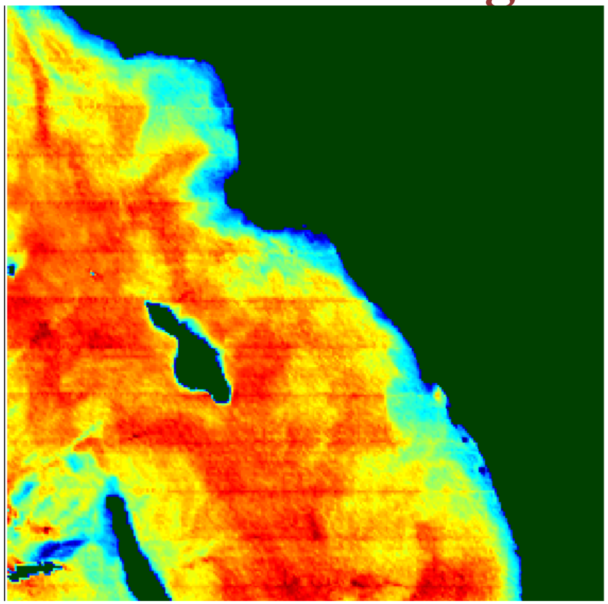
Affects

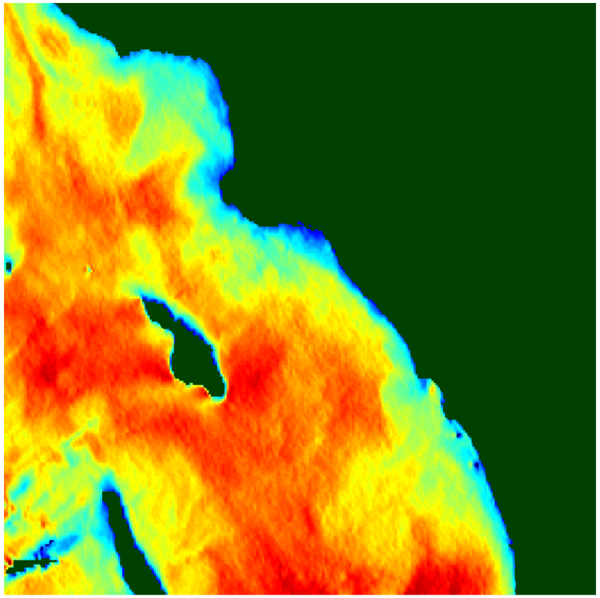
- The visual appearance
- Classification applications
- Quantitative applications

Goal

To remove the stripes while preserving the original information of the image

Striped Image





Destriped Image

Striping

Ubiquitous because of multiple causes

- Detector calibration errors
- View angle variances (differences in atmospheric scattering, sensitivity to polarization)
- Component artifacts (e.g. mirrors)

Destriping Methods

On-orbit calibrations: Solar & Lunar
Uniform earth images (e.g. deserts)

Scene based: Usually not calibrated against known reference
Inverse problems

Minimizing Functionals

- The problem is to find u, from $Au = b$ when A and b are given
- Solve $\arg\min_u \|Au - b\|_2^2$ instead of $Au = b$
- Since this is $L^2(\Omega)$ space, the problem can be written as

$$\arg\min_{u \in C} J(u), \text{ where } J(u) = \int_\Omega (Au - b)^2 d\Omega$$

- In the minimization process, the gradient of the functional is computed using Gateaux derivative as

$$\nabla J(u) = \frac{d}{d\tau} J(u + \tau h)\Big|_{\tau=0}$$

- Gradient of the functional

$$\nabla J(u) = 2A^*(Au - b)$$

Define the Destripe Functional

- Stripes are horizontal and need smooth only in vertical direction
- Expects that there is no gradient of the difference in horizontal direction
- Let

f – Striped image

u – Destriped image

Ω – Image Domain

- Then the energy functional $E(u)$, is

$$E(u)=\int_{\Omega}\left(\frac{\partial}{\partial x}(u-f)\right)^2 d\Omega+\alpha\int_{\Omega}\left(\frac{\partial u}{\partial y}\right)^2 d\Omega$$

- Minimize the functional from a variational approach
- First compute the Euler-Lagrange equation

Minimize the Functional

- The corresponding Euler-Lagrange equation is

$$u_{xx} + \alpha u_{yy} = f_{xx},$$

where α is the regularization parameter

- Using the derivative operators, this can be re-written as

$$D_{xx}u + \alpha D_{yy}u = D_{xx}f$$

$$\Rightarrow \left[D_{xx} + \alpha D_{yy}\right]u = D_{xx}f$$

- Solve the linear system to uncover u

Note

★ Need to construct the derivative operators D_{xx} and D_{yy}

★ Select the best regularization parameter α

Derivative Operators

- Consider a matrix M of size 3×5 where we want to compute M_{xx} using a finite difference stencil

$$\frac{\partial^2 m_{ij}}{\partial x^2} = \frac{1}{12h^2}\left[-m_{ij-2} + 16m_{ij-1} - 30m_{ij} + 16m_{ij+1} - m_{ij+1}\right]$$

- With reflexive boundary conditions and writing the array in vector form

m_4	m_1	m_1	m_4	m_7	m_{10}	m_{13}	m_{13}	m_{10}
m_5	m_2	m_2	m_5	m_8	m_{11}	m_{14}	m_{14}	m_{11}
m_6	m_3	m_3	m_6	m_9	m_{12}	m_{15}	m_{15}	m_{12}

The boundary points are in red

- Resulting operator is a 15×15 sparse array

Derivative Operators D_{xx}

$$\frac{1}{12h^2}\times$$

m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	
-14	0	0	15	0	0	-1	0	0	0	0	0	0	0	0	m_1
0	-14	0	0	15	0	0	-1	0	0	0	0	0	0	0	m_2
0	0	-14	0	0	15	0	0	-1	0	0	0	0	0	0	m_3
15	0	0	-30	0	0	16	0	0	-1	0	0	0	0	0	m_4
0	15	0	0	-30	0	0	16	0	0	-1	0	0	0	0	m_5
0	0	15	0	0	-30	0	0	16	0	0	-1	0	0	0	m_6
-1	0	0	16	0	0	-30	0	0	16	0	0	-1	0	0	m_7
0	-1	0	0	16	0	0	-30	0	0	16	0	0	-1	0	m_8
0	0	-1	0	0	16	0	0	-30	0	0	16	0	0	-1	m_9
0	0	0	-1	0	0	16	0	0	-30	0	0	16	0	0	m_{10}
0	0	0	0	-1	0	0	16	0	0	-30	0	0	16	0	m_{11}
0	0	0	0	0	-1	0	0	16	0	0	-30	0	0	16	m_{12}
0	0	0	0	0	0	-1	0	0	15	0	0	-14	0	0	m_{13}
0	0	0	0	0	0	0	-1	0	0	15	0	0	-14	0	m_{14}
0	0	0	0	0	0	0	0	-1	0	0	15	0	0	-14	m_{15}

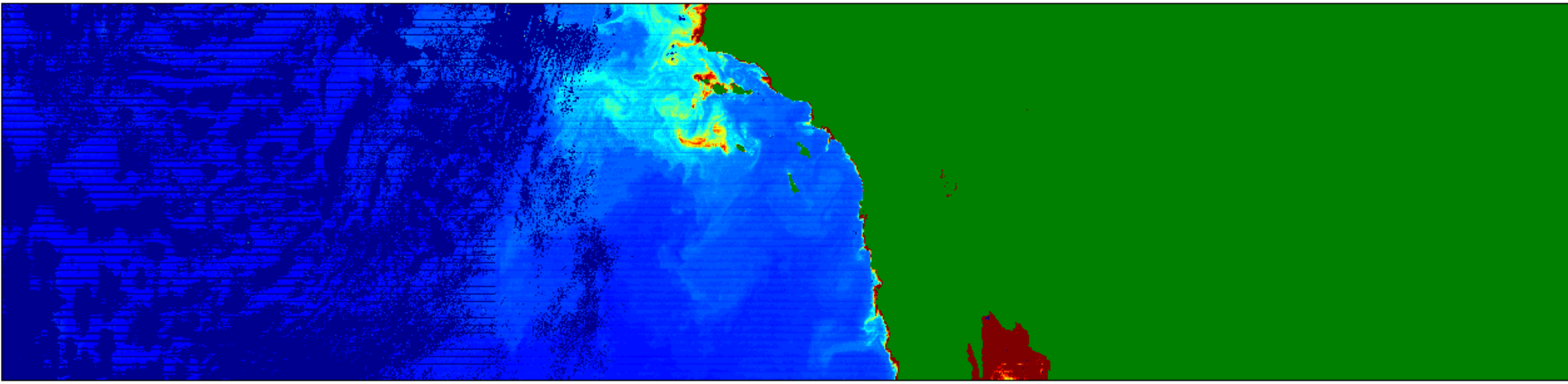
VIIRS Images

- Image covers the area surrounded by

  -122.09^0 W to -11.90^0 E, and 34.2^0 N to 31.6^0 S
- Near the Santa Monica region in Southern California

Earth Map

VIIRS Images

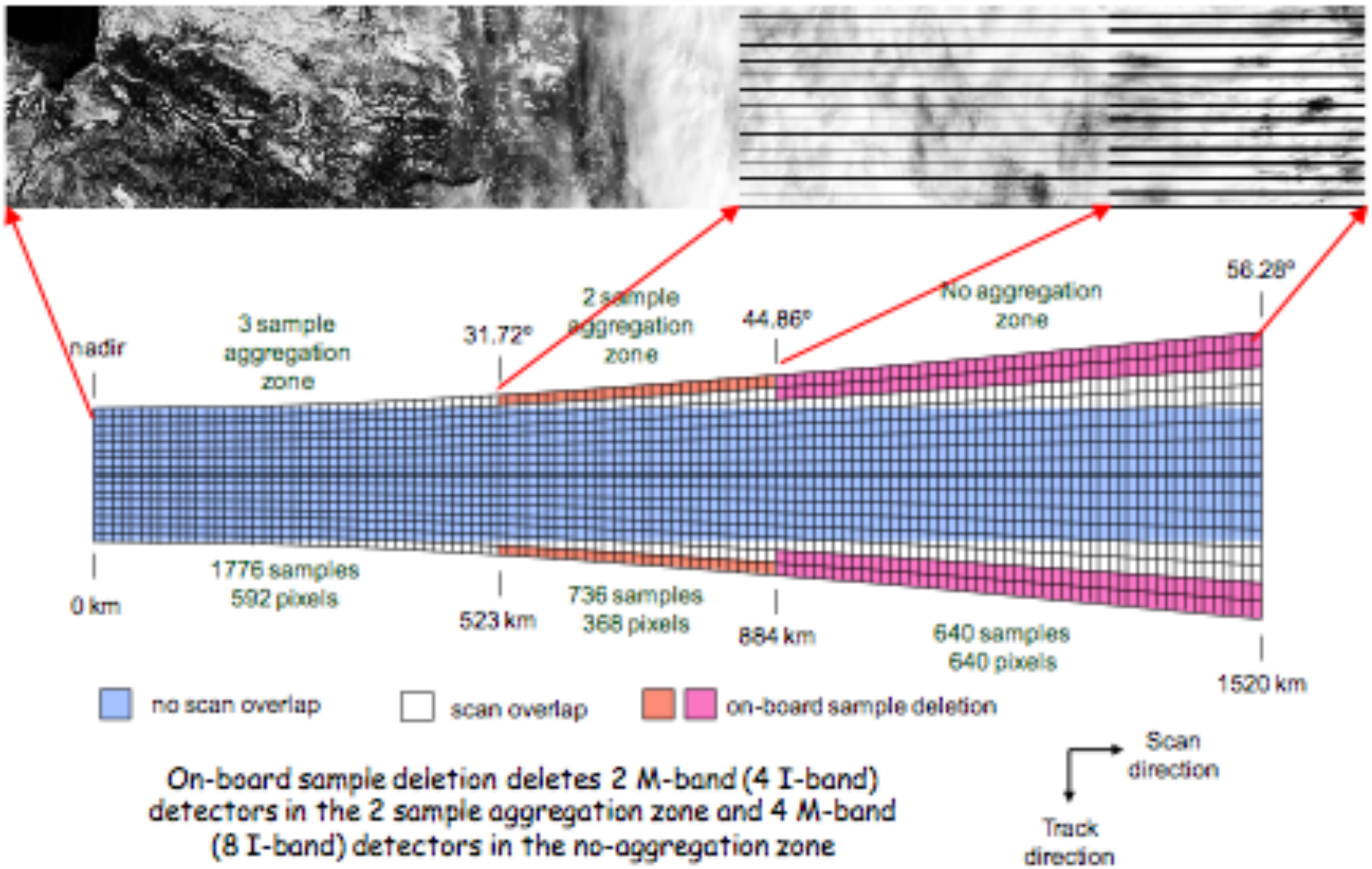


- The image was taken on November 6th, 2013
- Image Intensity represents Chlorophyll concentration
- **Green** - Land
- **Dark Blue** – Missing data

  Clouds and "Bow-tie" effects (Stripes)
- Stripes – Due to calibration errors between detectors

Bow-tie Effects

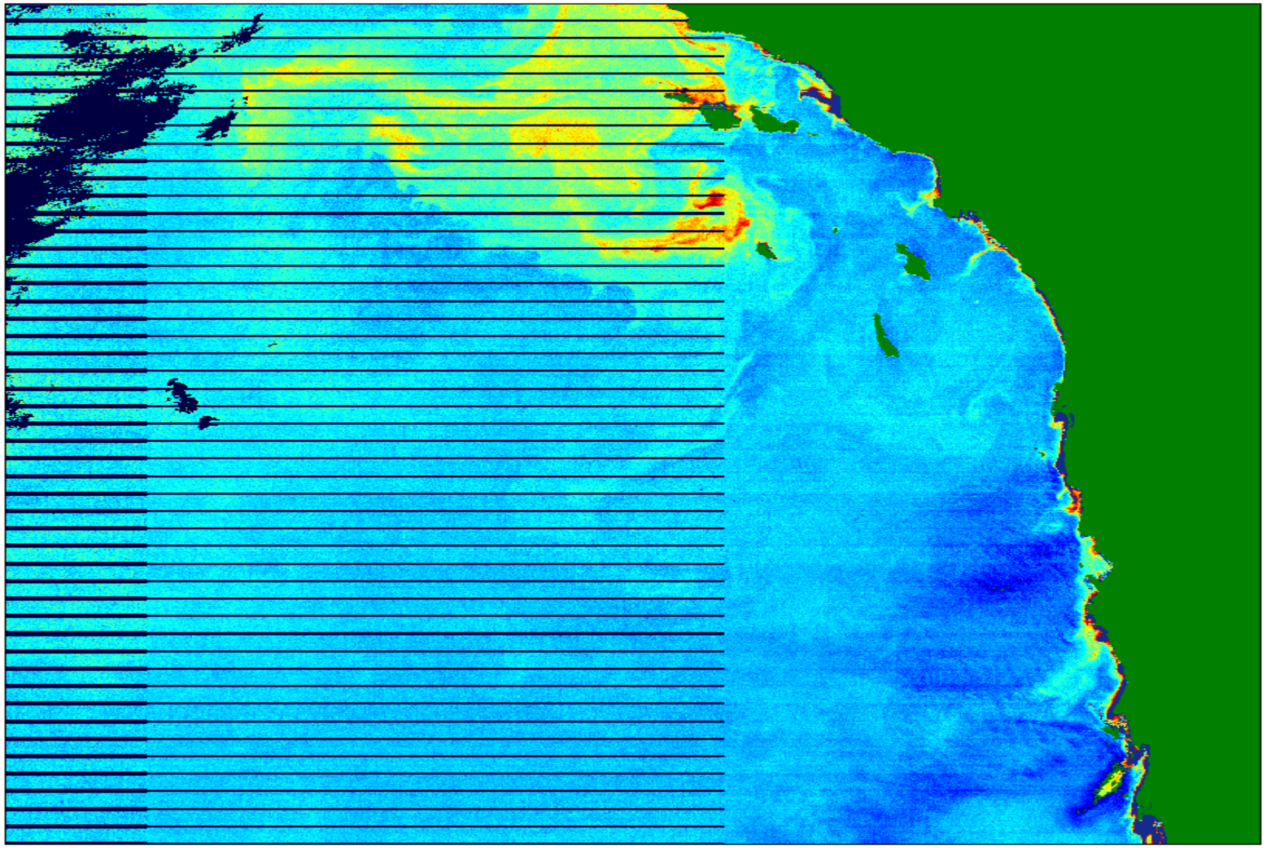
- When the view angle increases, consecutive detectors have overlap pixels
- Delete the overlap pixels during the transmission ('Bow-tie')



http://www.star.nesdis.noaa.gov/smcd/spb/nsun/snpp/VIIRS/VIIRS_SDR_Users_guide.pdf

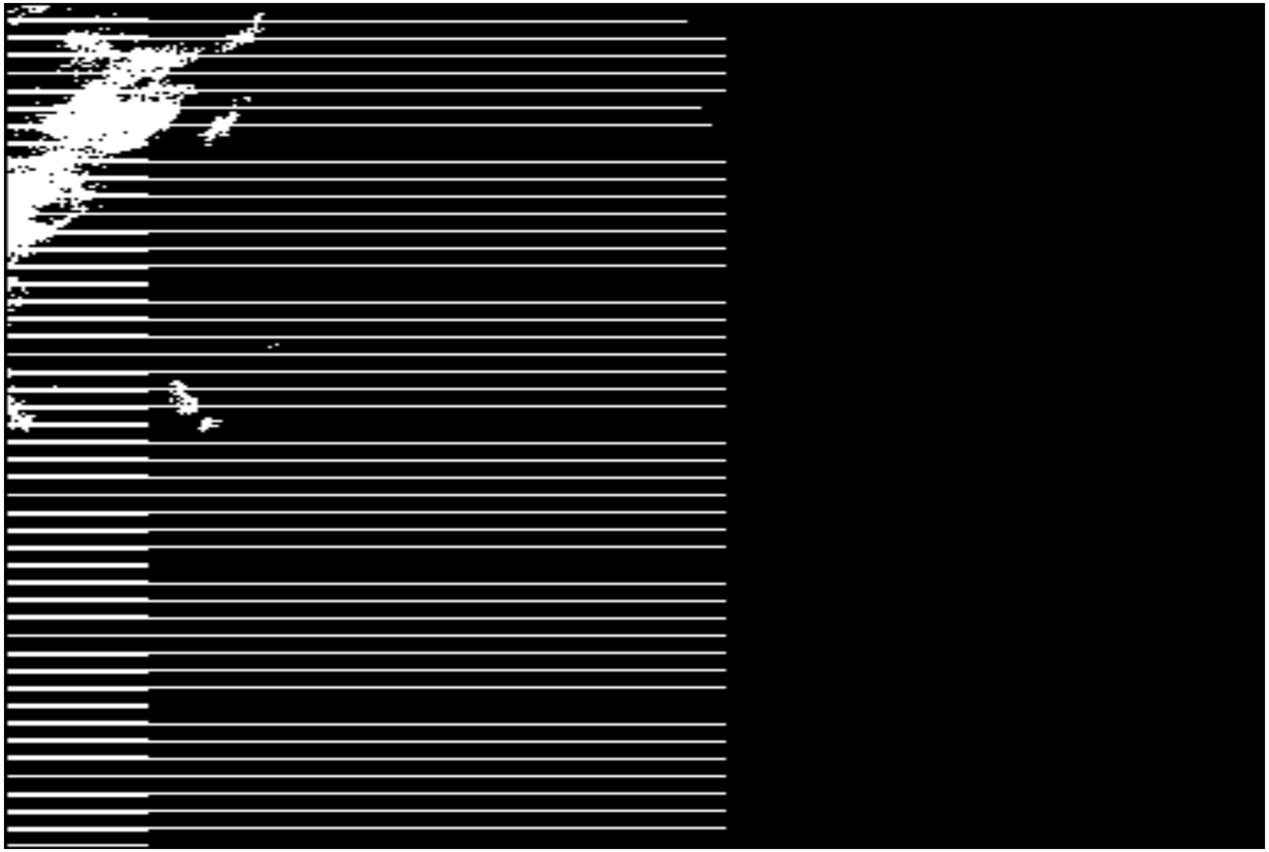
Inpaint Images

- Approximate the missing data due to clouds and bow-tie effects as destriping algorithm performs well on smooth images
- Use a technique called ‘inpainting’ use in image processing
- Consider a subsample of an VIIRS image on November 6th, 2013



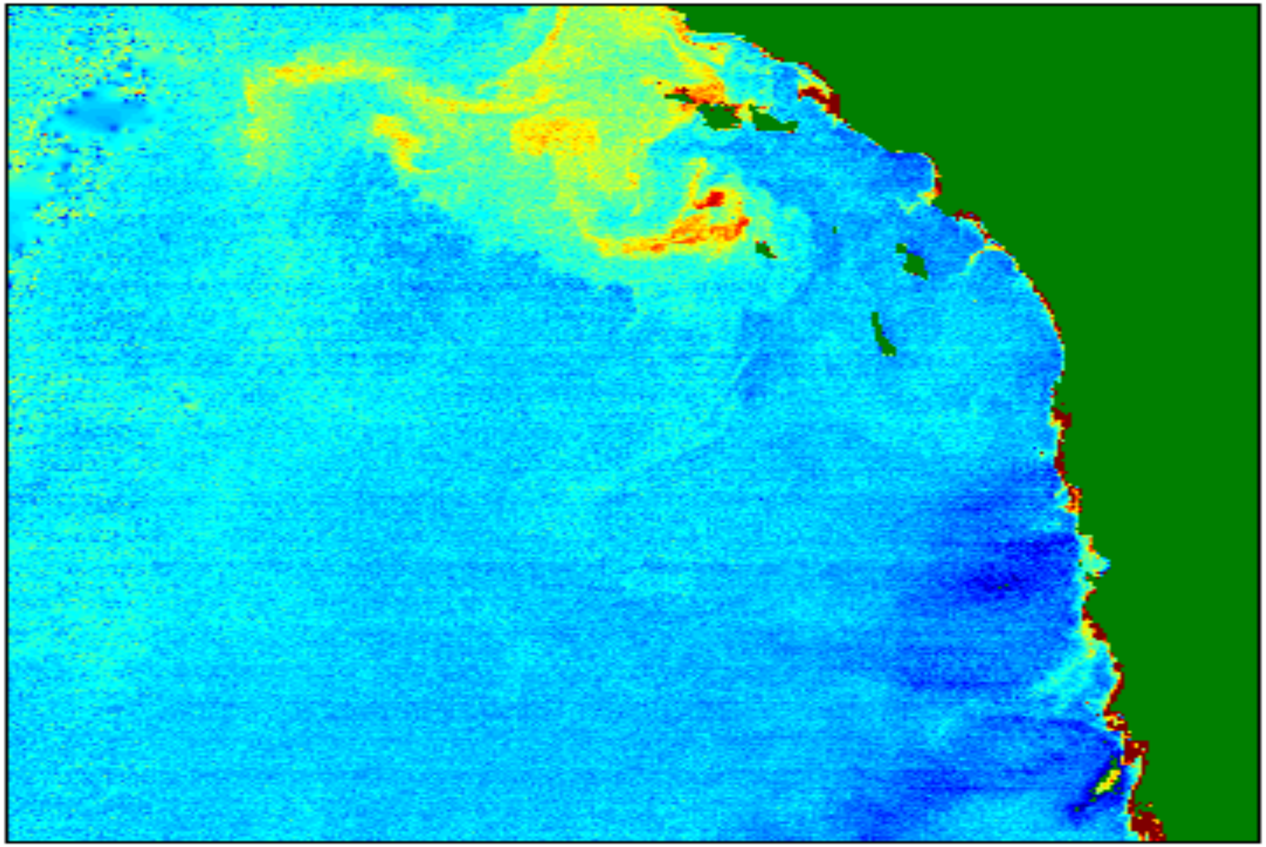
Inpaint Images

- Determine the mask to be inpainted
- Use 'im2bw' to extract the mask of the missing data
- White points – missing data
- Black points – good observations



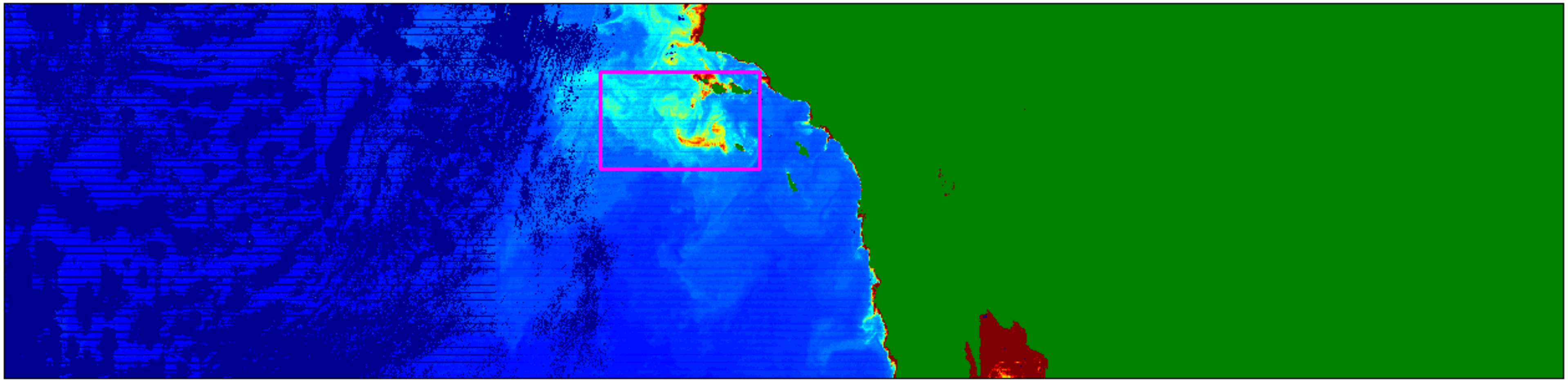
Inpaint Images

- Fill the missing data
- Use 'roifill' to estimate missing data



Destripe VIIRS Images

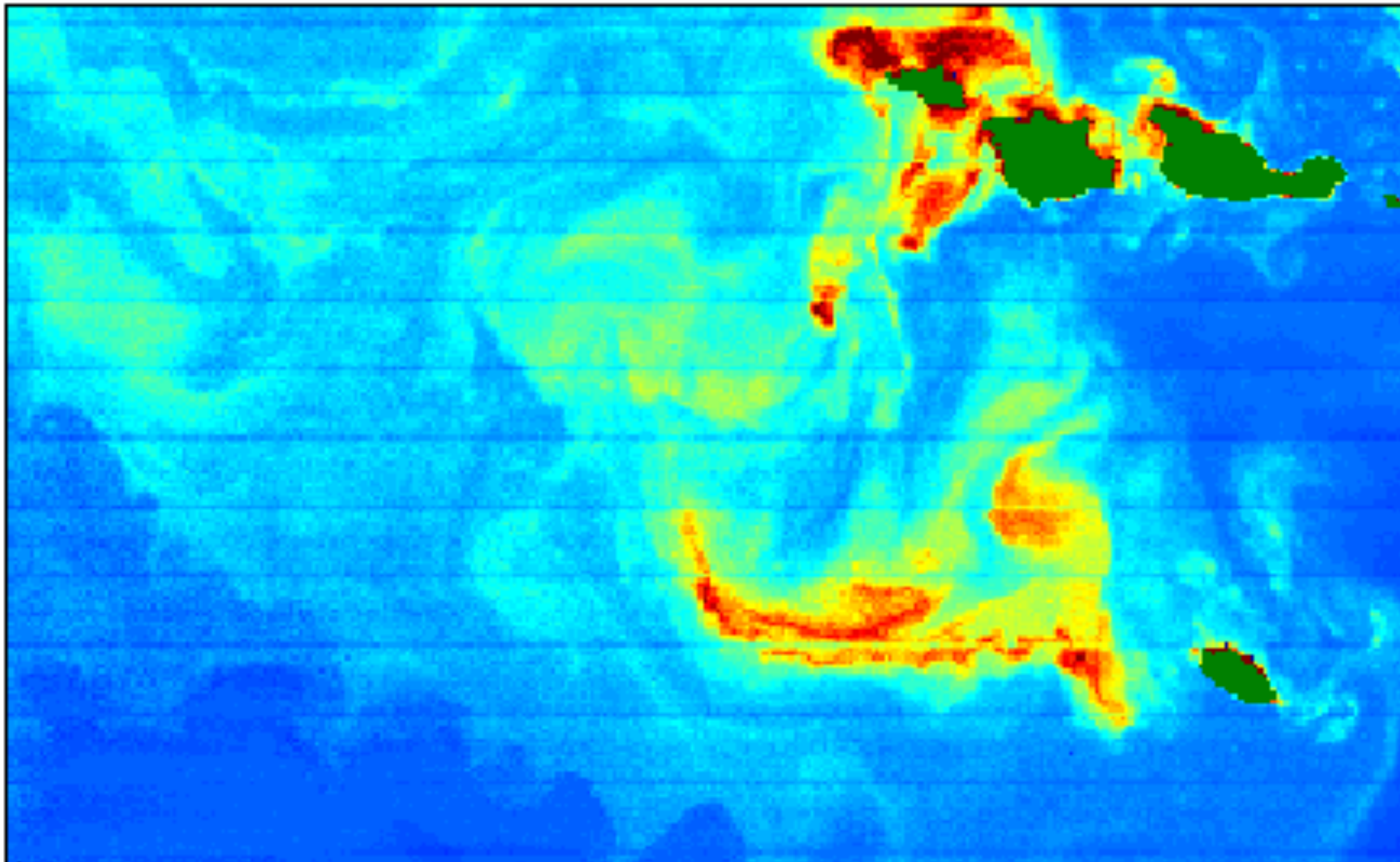
- Consider an image near the Santa Monica region in Southern California on November 6th, 2013



- Subsample the image as shown in pink rectangular domain to visualize the stripes properly

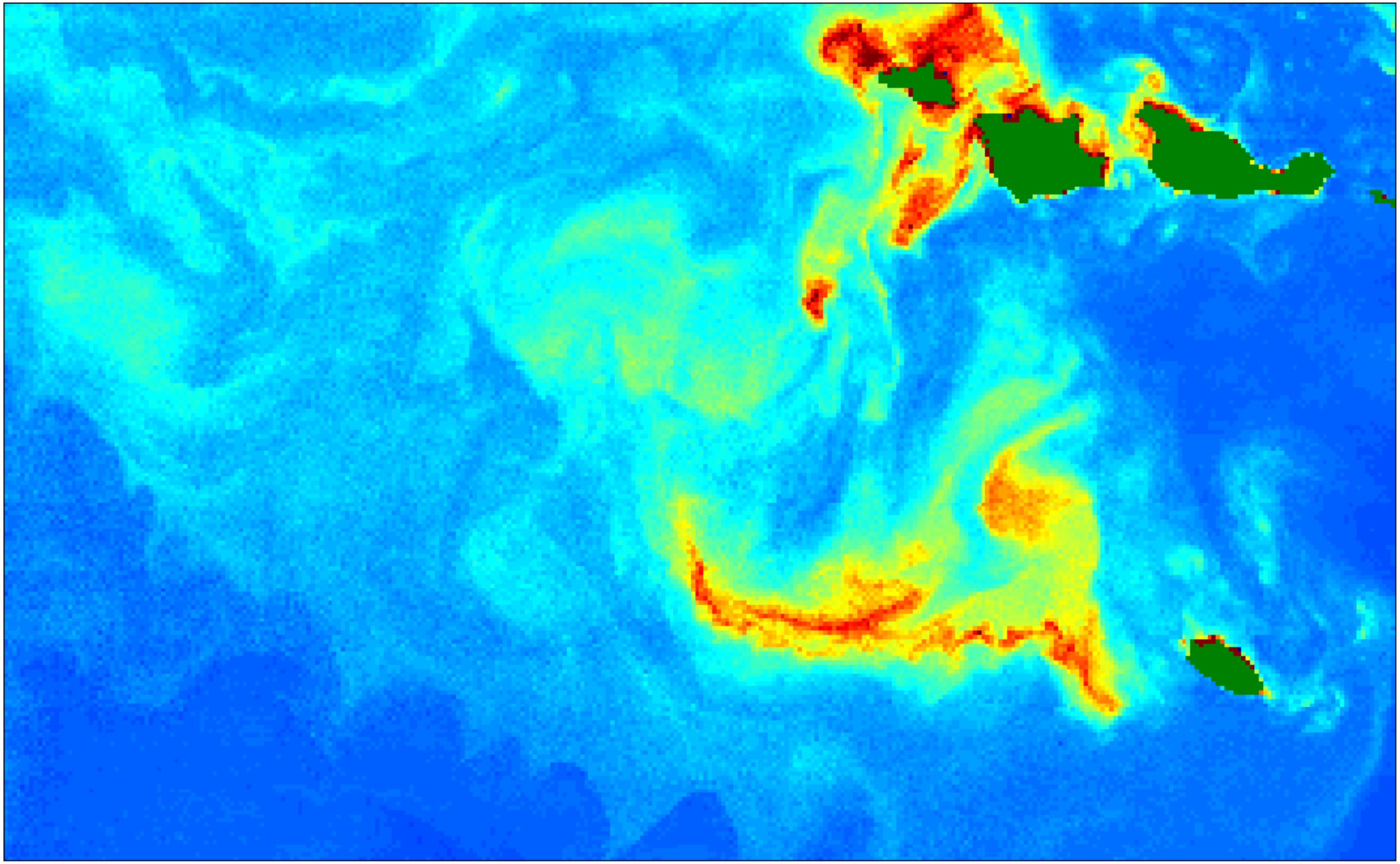
Destripe VIIRS Images

- Subsampled image near the Santa Monica region in Southern California on November 6th, 2013



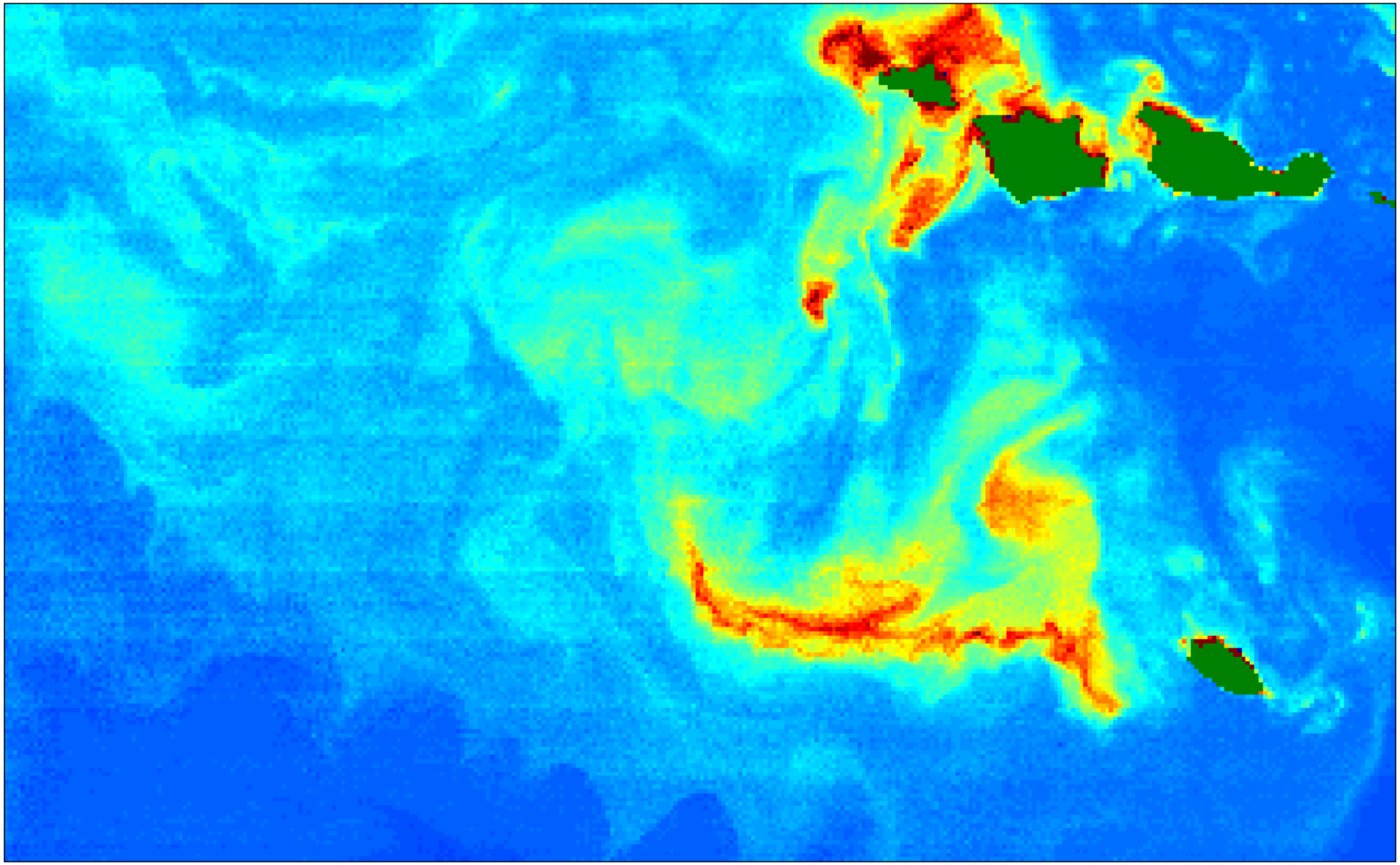
Destripe VIIRS Images

- After applying the destriping algorithm with reflexive boundaries and $\alpha = 10^{-4}$ on the subsampled image



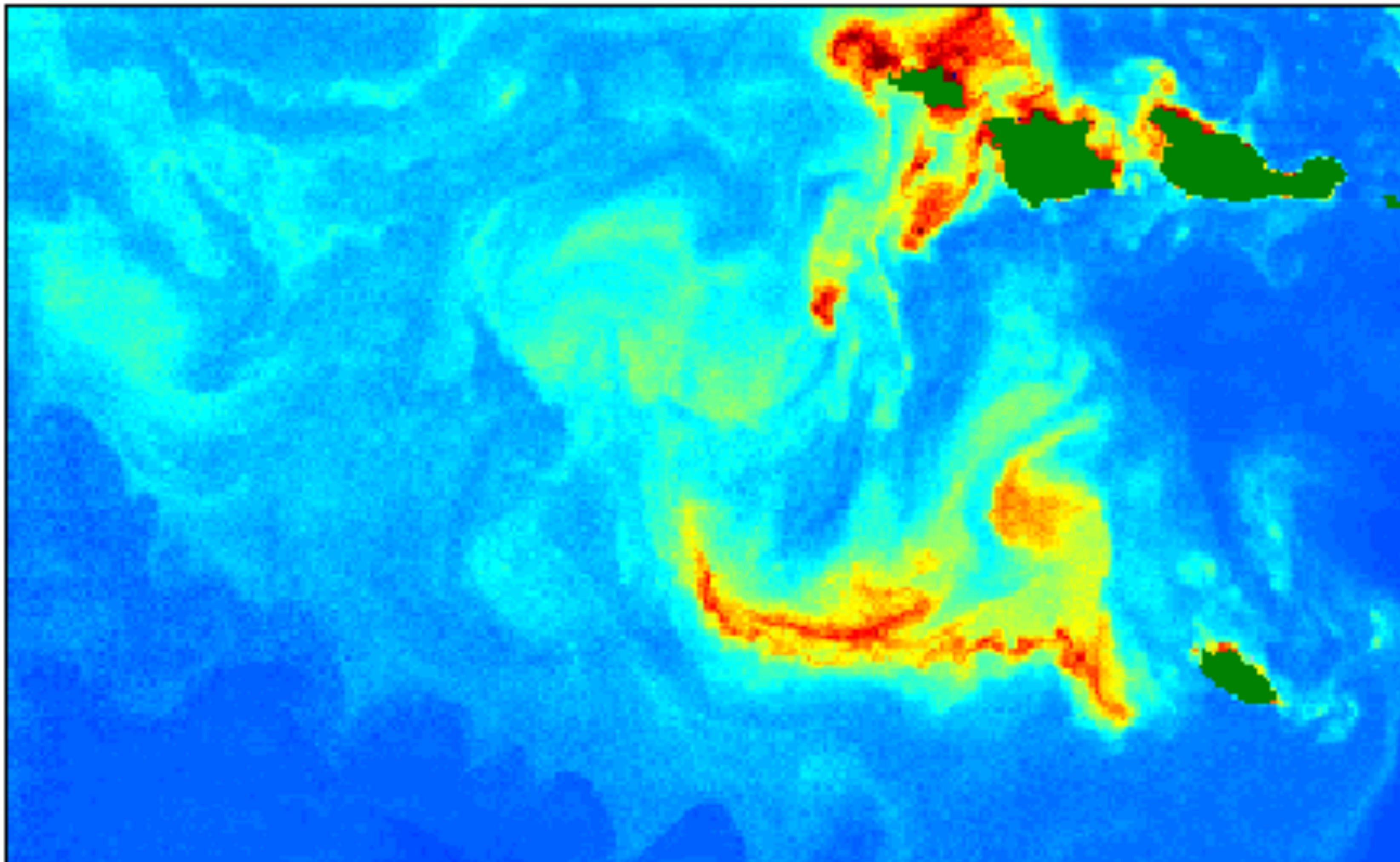
NASA destriped VIIRS Images

- Destriped image after NASA correction



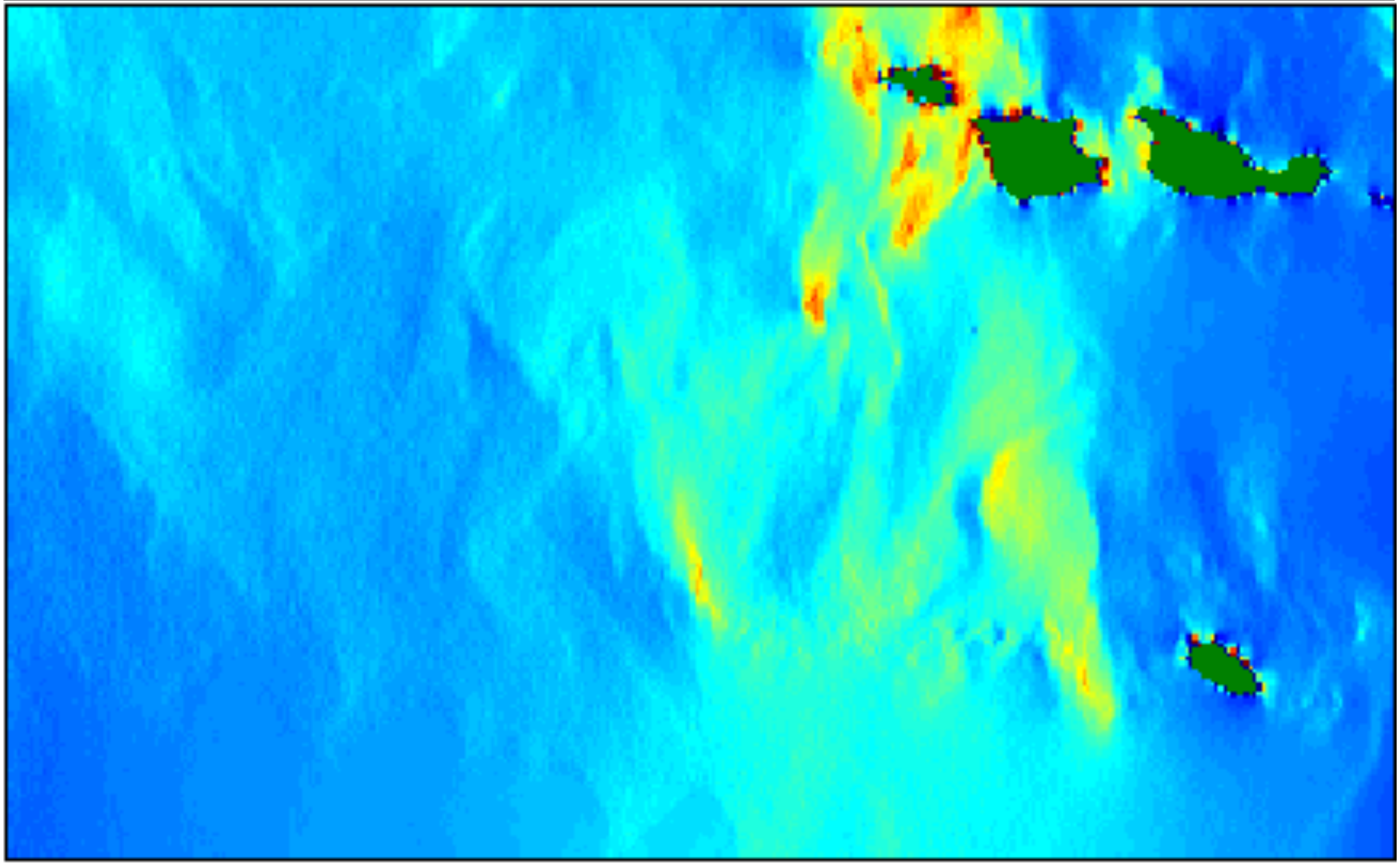
Destriping on NASA Destriped Image

- Destriped NASA corrected image with $\alpha = 8 \times 10^{-5}$



Destripe VIIRS Images with Alpha = 1

- After applying the destriping algorithm with reflexive boundaries and $\alpha = 1$ on the subsampled image



Selection of the Regularization Parameter

- Popular methods are:
  - The discrepancy principle
  - Generalized cross validation method
  - L-curve method and
  - U-curve method

U-curve method

- Consider the minimization problem

$$J(u) = \arg\min_u \|Au - z\|_2^2 + \alpha \|Lu\|_2^2$$

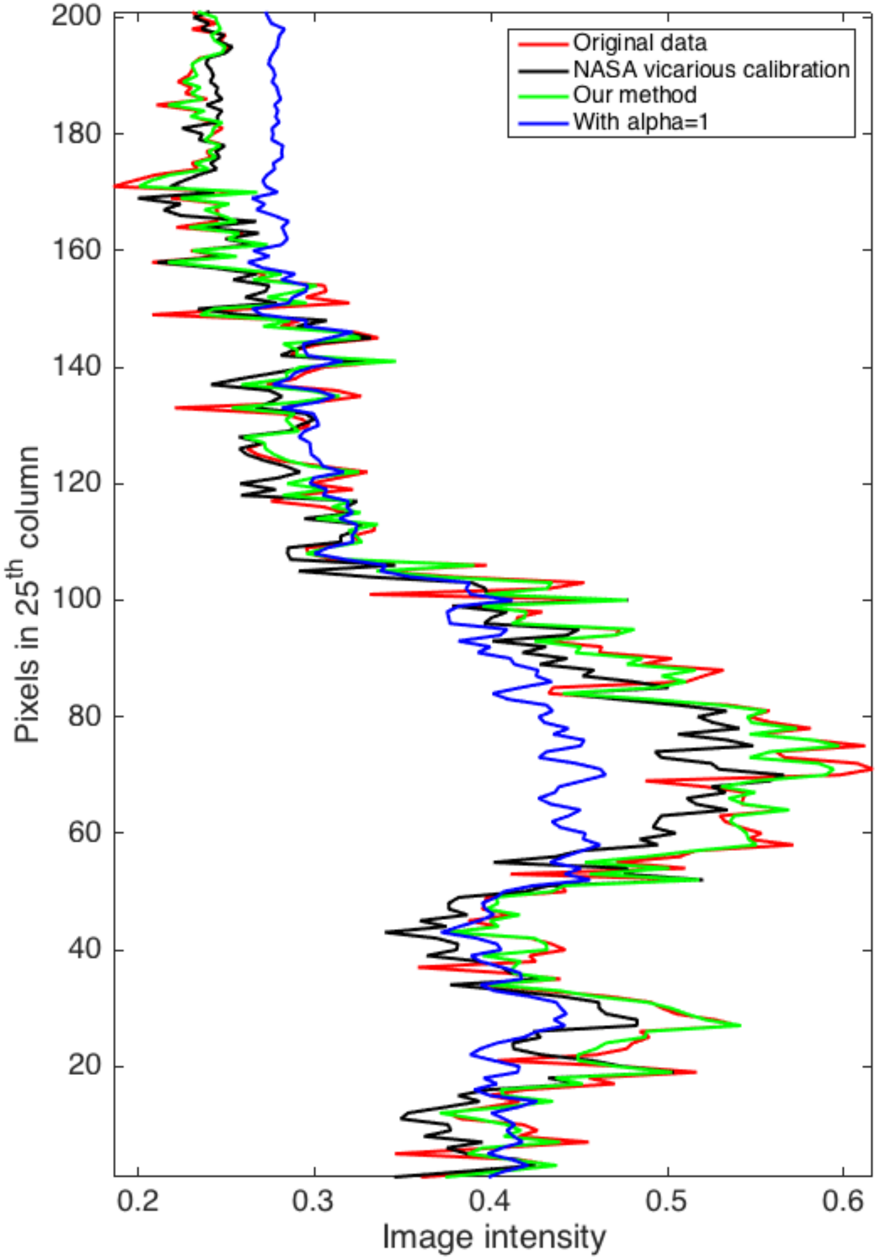
- Let $x(\alpha) = \|Au_\alpha - z\|_2^2$ and

$$\mathrm{y}(\alpha) = \|Lu_\alpha\|_2^2$$

- The best α is the minimizer of

$$U(\alpha) = \frac{1}{x(\alpha)} + \frac{1}{y(\alpha)}$$

Destriping Comparison



Multi Spectral - Destriping

For n=2

- Assume that the intensity patterns of two nearby spectrals are close to each other.
- Give a weighting factor β to avoid getting same destripe image from both spectral

$$E(u_1,u_2) \;= \int_\Omega \left(\frac{\partial}{\partial x}(u_1 - I_1)\right)^2 d\Omega + \alpha_1 \int_\Omega \left(\frac{\partial u_1}{\partial y}\right)^2 d\Omega$$

$$+\beta \int_\Omega (u_1 - u_2)^2 d\Omega + \int_\Omega \left(\frac{\partial}{\partial x}(u_2 - I_2)\right)^2 d\Omega + \alpha_2 \int_\Omega \left(\frac{\partial u_2}{\partial y}\right)^2 d\Omega$$

Where, I – the striped spectrum and u – the destriped spectrum

Euler Lagrange equations are

$$\frac{\partial^2 u_1}{\partial x^2} + \alpha \frac{\partial^2 u_1}{\partial y^2} + \beta(u_1 - u_2) = \frac{\partial^2 I_1}{\partial x^2}$$

$$\frac{\partial^2 u_2}{\partial x^2} + \alpha \frac{\partial^2 u_2}{\partial y^2} - \beta(u_1 - u_2) = \frac{\partial^2 I_1}{\partial x^2}$$

Multi Spectral - Destriping

For n>2

- The generalized energy functional can be obtained as,

$$E(u_1, u_2, \dots, u_n) = \sum_{i=1}^{n}\left(\int_{\Omega}\left(\frac{\partial}{\partial x}(u_i - I_i)\right)^2 d\Omega\right) + \sum_{i=1}^{n}\left(\alpha_i \int_{\Omega}\left(\frac{\partial u_i}{\partial y}\right)^2 d\Omega\right)$$

$$+\beta \sum_{i=1}^{n-1}\int_{\Omega}(u_i - u_{i+1})^2\, d\Omega$$

The system of Euler Lagrange equations are

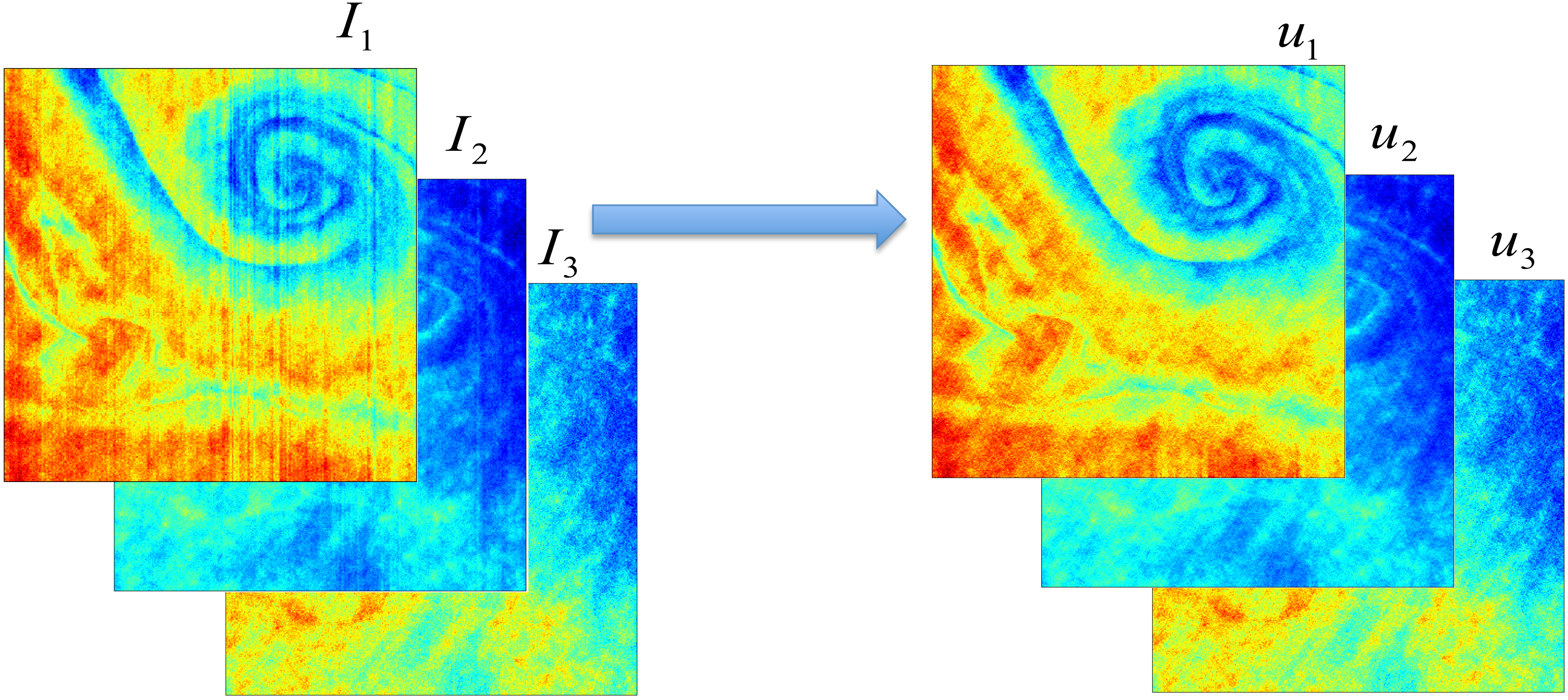
$$For\ k = 1,\ \frac{\partial^2 u_1}{\partial x^2} + \alpha_1 \frac{\partial^2 u_1}{\partial y^2} + \beta(u_1 - u_2) = \frac{\partial^2 I_1}{\partial x^2}$$

$$For\ k = n, \frac{\partial^2 u_n}{\partial x^2} + \alpha_n \frac{\partial^2 u_n}{\partial y^2} - \beta(u_{n-1} - u_n) = \frac{\partial^2 I_n}{\partial x^2}$$

$$For\ k = 2,3, \dots n-1, \frac{\partial^2 u_k}{\partial x^2} + \alpha_k \frac{\partial^2 u_k}{\partial y^2} + \beta(-u_{k-1} + 2u_k - u_{k+1}) = \frac{\partial^2 I_k}{\partial x^2}$$

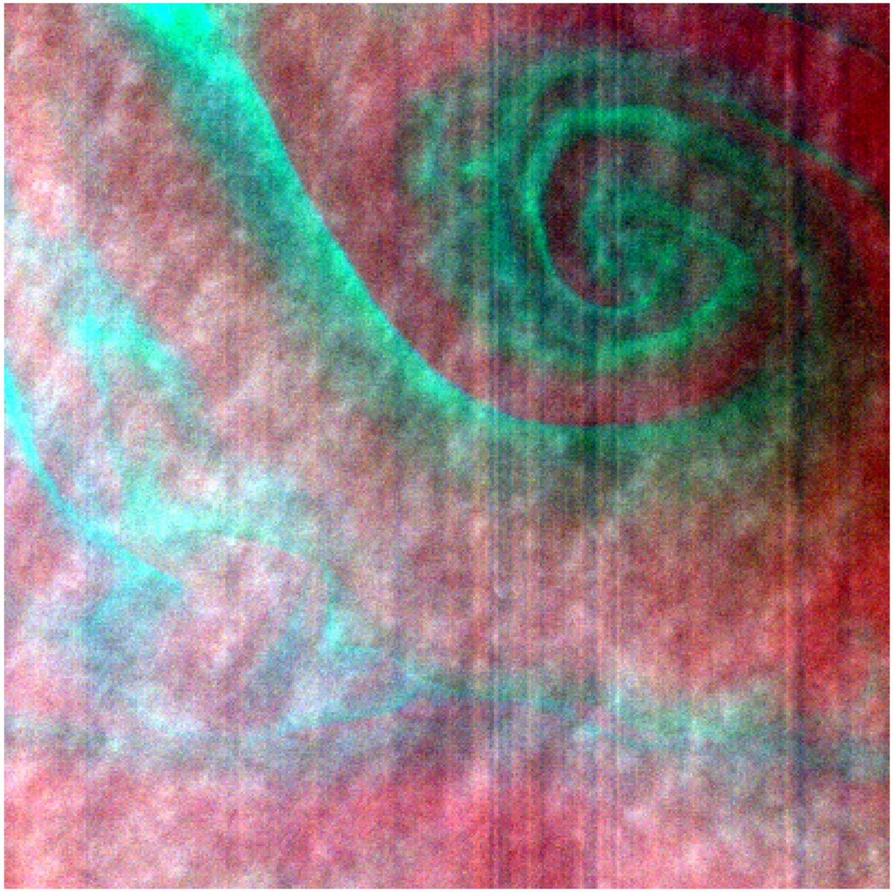
Spectral-wise Representation

- Destripe three spectral at once
- Wavelengths are 450 nm, 559 nm and 696 nm

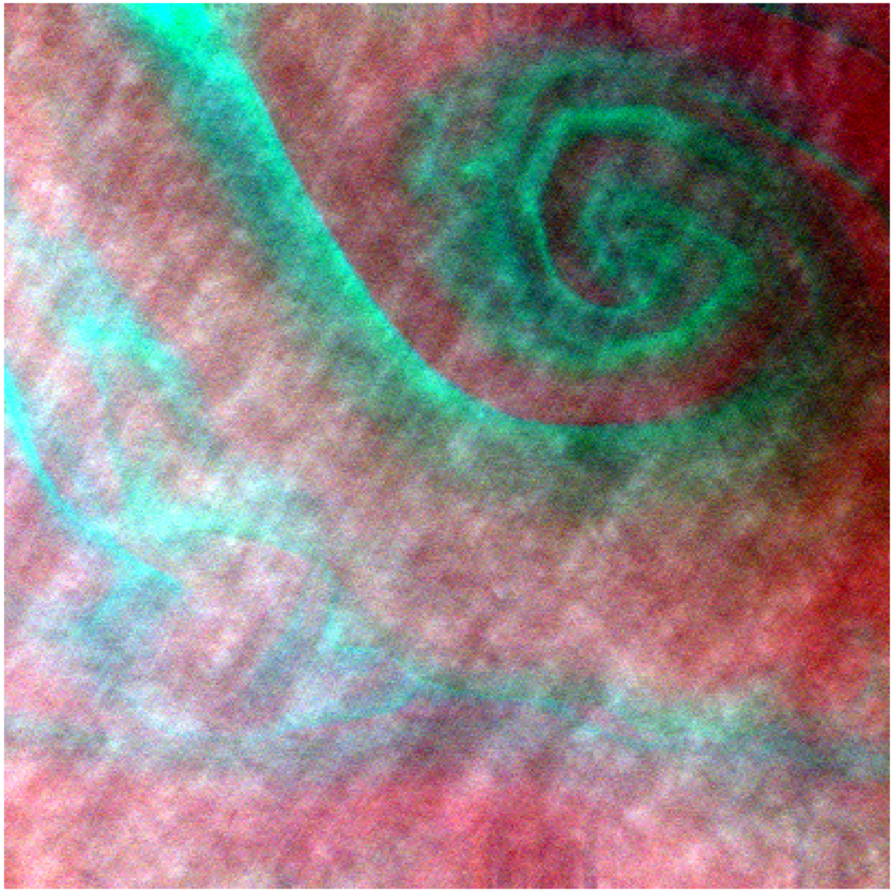


Multi Spectral – Destriping on HICO Images

- Combining three spectrals as an RGB image



Striped image



Destriped image

Conclusion and Future Work

- Solution of the destriped functional would be $u + c$ for any constant c
- Shift the mean of the destriped image to the mean of the striped image to obtain the appropriate solution
- Multi-spectral destriping is important when we do further analysis such as computation of optical flow fields from the destriped images.
- Selecting the regularization parameter is not consistent with the available standard methods
- Need to develop a new approach to avoid the difficulty of selecting regularization parameter.
- Use Statistical inverse problem techniques to estimate samples of destriped images

Inverse Problem

$$b = Au + \eta$$

$$A = [a_{ij}]_{m \times n} \in \mathbb{R}^{m \times n}$$ model (known)

$\boldsymbol{b} \in \mathbb{R}^m$ observed data

$\boldsymbol{\eta} \in \mathbb{R}^n$ noise (unknown)

$\boldsymbol{u} \in \mathbb{R}^n$ state vector (unknown)

Statement of the **inverse problem**
-- Given: model *A* and data ***b***
-- Goal: infer *u*, the unknown state vector

Bayesian approach (Statistical Inversion)

Given: form of the model $b = Au + \eta$

Express: the posterior distribution using the Bayes rule

$$p(u\,|\,b) = \frac{1}{p(b)}\, p(b\,|\,u) \cdot p(u)$$

normalization **constant**
(does **not** affect solutions)

prior distribution
(acts as "**regularization**")

"likelihood" function
(determined by the **model**)

Then: **estimate** solutions by sampling from $p(u\,|\,b)$
Sample Priors using MCMC method

A. Gelman, J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, and D. B. Rubin, *Bayesian Data Analysis, Third Edition*, Chapman & Hall/CRC, 2014.

J. Kaipio and E. Somersalo, *Statistical and Computational Inverse Problems*, Springer 2005.

Thank you