Parametric instability of absolute and relative motion of point-vortex systems in a multi-layer flow under linear deformation

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Systematic studies of point vortex systems started in the middle on the nineteenth century with the works of H. Helmholtz and G. Kirchhoff. Nevertheless, the point vortex systems have still been drawing a lot of attention since they find applications in a vast spectrum of physical problems. Here we consider a dynamical system governing the motion of many point vortices located in the different layers of a multi-layer flow under influence of an external deformation field. We show that the vorticity center of the system behaves just as the one of two point vortices interacting in a deformation flow. In particular, the vorticity center may experience parametric instability leading to its unbounded growth under the influence of non-stationary shear and rotational components oscillating with different magnitudes. We then show that, in a reference frame moving with vorticity center, the equation of relative vortex motion look the same as if the vorticity center stays in the origin of the reference frame. In addition, we study the relative motion of two point vortices located in different layers of a two-layer deformation flow. We analyze phase portraits of the relative motion and establish the conditions leading to parametric instability of vortex trajectories near the elliptic points. Also, we establish that the relative vortex motion can be chaotic due to a non-stationary perturbation from the background flow.